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Soft Binary Piecewise Theta Operation: A New Operation for Soft Sets

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Abstract: *After being presented by Molodtsov in 1999, soft set theory became well-known as a novel strategy for resolving uncertainty-related issues and modeling uncertainty. It has several uses in both theoretical and real-world settings. In this study, a novel soft set operation known as the "soft binary piecewise theta operation" is presented. Its fundamental algebraic properties are investigated in detail. Furthermore, the distributions of this operation over other soft set operations are examined. In addition to being a right-left system under certain circumstances, we demonstrate that the soft binary piecewise theta operation is also a commutative semigroup in the collection of soft sets over the universe. Furthermore, by taking into account the algebraic properties of the operation and its distribution rules together, we demonstrate that the collection of soft sets over the universe, along with the soft binary piecewise theta operation and some other types of soft sets, form many important algebraic structures, like semirings and near-semirings.*

Keywords: *Soft sets, Soft set operations, Conditional complements, Soft binary piecewise theta operation*

1. Introduction

Some of the theories that may be used to describe uncertainty include probability theory, interval mathematics, and fuzzy set theory; however, each of these theories has drawbacks of its own. First introduced by Molodtsov (1999), Soft Set Theory is a novel method for representing uncertainty and applying it to the resolution of problems pertaining to uncertainty. Since its introduction, this theory has been effectively applied to a number of mathematical areas. Among these are the fields of measurement theory, game theory, probability theory, Riemann integration, and Perron integration.

The initial research on soft set operations was done by Pei and Miao (2005) and Maji et al. (2003). Many soft set operations, such as restricted and extended soft set processes, were proposed by Ali et al. (2009). Sezgin and Atagün (2011) established and provided properties of the restricted symmetric difference of soft sets in their work on soft sets. They also went over the foundations of soft set operations and demonstrated how they relate to one another. Ali et al. (2011) conducted a comprehensive analysis of the algebraic structures of soft sets. Soft set operations piqued the interest of several scholars for many years. The notion of the soft binary piecewise difference operation in soft sets was proposed by Eren and Çalışıcı (2019). Additionally, a detailed investigation of the soft binary piecewise difference operation was conducted by Sezgin and Çalışıcı (2024). The extended difference of soft sets was first presented by Sezgin et al. (2019), and Stojanovic (2021) defined and examined the characteristics of the extended symmetric difference of soft sets.

Çağman (2001) brought two new complement operations to the literature, Sezgin et al. (2023a) worked on these new binary set operations and reported numerous more. By applying these new binary operations to soft sets, Aybek (2024) established a large number of additional restricted and extended soft set operations. In their ongoing effort to alter the structure of extended operations in soft sets, Akbulut (2024), Demirci (2024), and Sarıalioğlu (2024) focused on the complementary extended soft set operations. By significantly changing the form of the soft binary piecewise operation in soft sets, comlementary soft binary piecewise operations were studied by Sezgin and Demirci (2023), Sezgin et al. (2023b, 2023b), Sezgin and Yavuz (2023a), Sezgin and Dagtoros (2023), Sezgin and Çağman (2024), Sezgin and Sarıalioğlu (2024a). Studies of soft sets as regards the abstract algebra has also piqqued attention by the researchers for many years. We refer to (Aktaş and Çağman, 2007; Jun, 2008; Jun and Park, 2008; Park et al., 2008; Feng et al., 2008; Sun et al., 2008; Acar et al., 2010; Zhan and Jun, 2010; Sezer et al., 2013, Sezer et al., 2014; Atagün and Sezgin, 2015; Atagün and Sezer, 2015; Sezer et al; Muştuoğlu et al., 2016; Mahmood et al., 2015; Sezer and Atagün, 2016; Tunçay and Sezgin, 2016; Sezgin, 2016; Sezer et al., 2017; Atagün and Sezgin, 2017; Khan et al., 2017; Sezgin et al., 2017; Ullah et al., 2018; Iftikhar and Mahmood, 2018; Atagün and Sezgin, 2018; Gulistan et al., 2018; Sezgin, 2018; Jana et al., 2019; Atagün et al., 2019; Karaaslan, 2019; Özlü and Sezgin, 2020; Karaaslan et al., 2021; Sezgin et al., 2022; Sezgin and Orbay, 2022; Manikantan et al., 2023

As an extension of rings, one of the most well-known concepts in binary algebraic structures are near-rings, semirings, and semifields. For a very long time, researchers have been interested in learning more about this subject. The term semirings was initially defined by Vandiver (1934). The seminearring (near-semiring) was discussed by Hoorn and Rootselaar (1967).

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In this paper, we want to make a major contribution to the field of soft set theory by introducing the "soft binary piecewise theta operation" and closely examining the algebraic structures associated with it as well as other soft set operations in the soft set collections. The structure of this study is as follows: The fundamental concepts of soft sets and various algebraic structures are reviewed in Section 2. In the third section, the algebraic characteristics of the newly proposed soft set operation are analyzed in detail. These characteristics enable us to demonstrate that, in addition to being a right-left system with the right identity empty soft set under specific circumstances, the soft binary piecewise theta operation is a commutative semigroup.

The distribution of the soft binary piecewise theta operation over various soft set operations, including restricted, extended, and soft binary piecewise operations, is examined in Section 4. A thorough examination of the algebraic structures produced by the collection of soft sets with these operations is given, taking into account the distribution laws and the algebraic characteristics of the soft set operations. It is shown that the collection of soft sets over the universe with the soft binary piecewise theta operation and other types of soft sets construct various important algebraic structures, including semirings and near-semirings. The importance of the study's findings and their possible relevance to the topic are covered in Section 5.

2. Preliminaries

This section summarizes and provides several algebraic structures as well as some basic concepts in soft set theory.

Definition 2.1. Let U be the universal set, E be the parameter set, P(U) be the power set of U, and let $K \subseteq E$. A pair (F, K) is called a soft set on U. Here, F is a function given by $F: K \rightarrow P(U)$ (Molodtsov, 1999).

The set of all soft sets over U is denoted by $S_E(U)$. Let K be a fixed subset of E, then the set of all soft sets over U with the fixed parameter set K is denoted by $S_K(U)$.

Definition 2.2. Let (F,K) be a soft set over U. If $F(e)=\emptyset$ for all $e \in K$, then the soft set (F,K) is called a null soft set with respect to K, denoted by $\emptyset_K$. Similarly, let (F,E) be a soft set over U. If $F(e)=\emptyset$ for all $e \in E$, then the soft set (F,E) is called a null soft set with respect to E, denoted by $\emptyset_E$ (Ali et al., 2011).

It is known that a function $F: \emptyset \longrightarrow K$, where the domain is the empty set, is referred to as the empty function. Since the soft set is also a function, it is evident that by taking the domain as $\emptyset$, a soft set can be defined as $F: \emptyset \longrightarrow P(U)$, where U is a universal set. Such a soft set is called an empty soft set and is denoted as $\emptyset_\emptyset$. Thus, $\emptyset_\emptyset$ is the only soft set with an empty parameter set (Ali et al., 2009).

Definition 2.3. Let (F,K) be a soft set over U. If F(e)=U for all e∈K, then the soft set (F,K) is called an absolute soft set with respect to K, denoted by U_K. Similarly, let (F,E) be a soft set over U. If F(e)=U for all e∈E, then the soft set (F,E) is called an absolute soft set with respect to E, denoted by U_E (Ali et al., 2009).

Definition 2.4. Let (F,K) and (G,Y) be soft sets over U. If K⊆Y and for all e∈K, F(e) ⊆G(e), then (F,K) is said to be a soft subset of (G,Y), denoted by $(F,K)\tilde{\subseteq}(G,Y)$. If (G,Y) is a soft subset of (F,K), then (F,K) is said to be a soft superset of (G,Y), denoted by $(F,K)\tilde{\supseteq}(G,Y)$. If $(F,K)\tilde{\subseteq}(G,Y)$ and $(G,Y)\tilde{\subseteq}(F,K)$, then (F,K) and (G,Y) are called soft equal sets (Pei and Maio, 2005).

Definition 2.5. Let (F,K) be a soft set over U. The soft complement of (F,K), denoted by $(F,K)^r=(F^r,K)$, is defined as follows: for all e∈K, $F^r(e)=U-F(e)$ (Ali et al., 2009).

For more about soft set opeartions, we refer to Sezgin and Yavuz (2024), Sezgin and Sarıalioğlu (2024b); about band, semilattice, and bounded semilattice to Clifford (1954); semiring and hemiring, to Vandiver (1934); lattice, Boolean algebra, De Morgan algebra, and Stone algebra, to Birkhoff (1967); about MV-algebra to Chang (1959). Regarding the graph applications and network analysis concerning possible soft set applications, we refer to Pant et al. (2024), and to Ali et al. (2015), Jan et al. (2020), Irfan Siddique et al. (2021), and Mahmood (2020) for bipolar soft sets, double framed soft sets and lattice ordered soft sets. Furthermore, for more about soft AG-groupoids, soft KU-algebras, and picture soft sets, see Khan et al. (2015), Gulistan and Shahzad (2014), Memiş (2022), and Naeem and Memiş (2023), respectively.

3. COMPLEMENTARY EXTENDED INTERSECTION OPERATION

This section presents a new soft set operation called soft binary piecewise theta. It also gives an example of the operation, examines its algebraic properties and how it relates to other soft set operations, looks at the distribution rules and algebraic structures it forms in the $S_E(U)$ set, and provides some very significant results.

Definition 3.1. Let (F,K) and (G,Y) be soft sets over U. The soft binary piecewise theta of (F,K) and (G,Y) is the soft set (H,K), denoted by, $(F,K)\underset{\theta}{\sim}(G,Y)=(H,K)$, where for all $\omega\in K$,

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$$H(\omega)=\begin{cases} F(\omega), & \omega\in K\text{-}Y \\ F'(\omega)\cap G'(\omega), & \omega\in K\cap Y \end{cases}$$

Example 3.2. Let $E=\{e_1,e_2,e_3,e_4\}$ be the parameter set $K=\{e_1, e_4\}$ and $Y=\{e_2, e_3, e_4\}$ be the subsets of E and $U=\{h_1,h_2,h_3,h_4,h_5,h_6\}$ be the initial universe set. Assume that (F,K) and (G,Y) are the soft sets over U defined as following:

$$(F,K)=\{(e_1, \{h_2,h_4,h_6\}), (e_4,\{h_1,h_2,h_5,h_6\})\}$$

$$(G,Y)=\{(e_2,\{h_1,h_2\}), (e_3,\{h_2,h_3,h_4,h_5\}),(e_4,\{h_2,h_3,h_5\})\}$$

Let $(F,K)\ \tilde{\theta}\ (G,Y)=(H,K)$, where for all $\omega\in K$,

$$H(\omega)=\begin{cases} F(\omega), & \omega\in K\text{-}Y \\ F'(\omega)\cap G'(\omega), & \omega\in K\cap Y \end{cases}$$

Here, $K=\{e_1,e_4\}$, $K-Y=\{e_1\}$, for all $\omega\in K-Y=\{e_1\}$, $H(\omega)=F(\omega)$ and so $H(e_1)=F(e_1)=\{h_2,h_4,h_6\}$, for all $\omega\in K\cap Y=\{e_4\}$, $H(\omega)=F'(\omega)\cap G'(\omega)$ and so $H(e_4)=F'(e_4)\cap G'(e_4)=\{h_3,h_4\}\cap\{h_1,h_4,h_6\}=\{h_4\}$.Thus,$(F,K)\tilde{\theta}(G,Y)=\{(e_1,\{h_2,h_4,h_6\}),(e_4,\{h_4\})\}$.

Theorem 3.3. Let (F, K), (G,Y), (H,D), (F, K), (G,Y), (H,D), K,T), and (L, T) be soft sets over U. Then,

1) The set $S_E(U)$ is closed under $\tilde{\theta}$. That is, when (F,K) and (G,Y) are two soft sets over U, then so is $(F,K)\ \tilde{\theta}\ (G,Y)$.

Proof: It is clear that $\tilde{\theta}$ is a binary operation in $S_E(U)$ and $S_K(U)$.

2) If $K\cap Y'\cap D=K\cap Y\cap D=\emptyset$, then $[(F,K)\ \tilde{\theta}(G,Y)]\ \tilde{\theta}\ (H,D)=(F,K)\ \tilde{\theta}\ [(G,Y)\ \tilde{\theta}\ (H,D)]$.

Proof: Let first handle the left hand side (LHS) of the equality and let $(F,K)\tilde{\theta}(G,Y)=(T,K)$, where for all $\omega\in K$,

$$T(\omega)=\begin{cases} F(\omega), & \omega\in K-Y \\ F'(\omega)\cap G'(\omega), & \omega\in K\cap Y \end{cases}$$

Let $(T,K)\tilde{\theta}(H,D)=(M,K)$, where for all $\omega\in K$,

$$M(\omega)=\begin{cases} T(\omega), & \omega\in K-D \\ T'(\omega)\cap H'(\omega), & \omega\in K\cap D \end{cases}$$

Hence,

$$M(\omega)=\begin{cases} F(\omega), & \omega\in (K-Y)-D=K\cap Y'\cap D' \\ F'(\omega)\cap G'(\omega) & \omega\in (K\cap Y)-D=K\cap Y\cap D' \\ F'(\omega)\cap H'(\omega), & \omega\in (K-Y)\cap D=K\cap Y'\cap D \\ [F(\omega)\cup G(\omega)]\cap H'(\omega), & \omega\in (K\cap Y)\cap D=K\cap Y\cap D \end{cases}$$

Let $(G,Y)\tilde{\theta}(H,D)=(K,Y)$, where for all $\omega\in Y$,

$$K(\omega)=\begin{cases} G(\omega), & \omega\in Y-D \\ G'(\omega)\cap H'(\omega), & \omega\in Y\cap D \end{cases}$$

Let $(F,K)\tilde{\theta}(K,Y)=(S,K)$, where for all $\omega\in K$,

$$S(\omega)=\begin{cases} F(\omega), & \omega\in K-Y \\ F'(\omega)\cap K'(\omega), & \omega\in K\cap Y \end{cases}$$

Hence,

$$S(\omega)=\begin{cases} F(\omega), & \omega\in K-Y \\ F'(\omega)\cap G'(\omega), & \omega\in K\cap (Y-D)=K\cap Y\cap D' \\ F'(\omega)\cap [G(\omega)\cup H(\omega)], & \omega\in K\cap (Y\cap D)=K\cap Y\cap D \end{cases}$$

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When considering K-Y in the S function, since K-Y=K∩Y', if ω∈Y', then ω∈D-Y or ω∈(Y∪D)'. Thus, if ω∈K-Y, then ω∈K∩Y'∩D' or ω∈K∩Y'∩D. Thus, M=S for K∩Y'∩D= K∩Y∩D=∅. That is, under suitable conditions, the operation $\tilde{\theta}$ is associative $S_E(U)$.

3) $[(F,K)\,\tilde{\theta}\,(G,K)]\,\tilde{\theta}\,(H,K) \neq (F,K)\,\tilde{\theta}\,[(G,K)\,\tilde{\theta}\,(H,K)]$.

Proof: It is obvious by the proof of (2).That is, for the soft sets whose parameter sets are the same, the operation $\tilde{\theta}$ is not associative.

4) $(F,K)\,\tilde{\theta}\,(G,Y) \neq (G,Y)\,\tilde{\theta}\,(F,K)$.

Proof: While the parameter set of the soft set of the LHS is K, the parameter set of the soft set of the RHS is Y. Thus, by the definition of soft equality,

$$(F,K)\,\tilde{\theta}\,(G,Y) \neq (G,Y)\,\tilde{\theta}\,(F,K).$$

But it is obvious that $(F,K)\,\tilde{\theta}\,(G,K)=(G,K)\,\tilde{\theta}\,(F,K)$. That is, while the operation $\tilde{\theta}$ is not commutative in $S_E(U)$, the operation $\tilde{\theta}$ is commutative in $S_K(U)$, where K⊆E is a fixed parameter set. Namely, $(F,K)\,\tilde{\theta}\,(G,K)=(G,K)\,\tilde{\theta}\,(F,K)$.

5) $(F,K)\,\tilde{\theta}\,(F,K)=(F,K)^r$.

Proof: Let $(F,K)\,\tilde{\theta}\,(F,K)=(H,K)$, where for all ω∈K,

$$H(\omega)=\begin{cases} F(\omega), & \omega\in K-K=\emptyset \\ F'(\omega)\cap F'(\omega), & \omega\in K\cap K=K \end{cases}$$

Thus, for all ω∈K, $H(\omega)=F'(\omega)\cap F'(\omega)=F'(\omega)$, thus $(H,K)=(F,K)^r$. That is, the operation $\tilde{\theta}$ is not idempotent in $S_E(U)$.

Theorem 3.3.1. By Theorem 3.3 (1), (2) ve (4), $(S_E(U),\tilde{\theta})$ is a noncommutative and not idempotent semigroup, under the condition K∩Y'∩D=K∩Y∩D=∅, where (F,K), (G,Y) and (H,D) are elements of $S_E(U)$. By Theorem 3.3. (3) since $\tilde{\theta}$ is not associative in $S_K(U)$, where K⊆ E is a fixed parameter set, $(S_K(U),\tilde{\theta})$ is not a semigroup, however, it is obvious that it is a commutative groupoid.

6) $(F,K)\underset{\theta}{\sim}\emptyset_K=\emptyset_K\underset{\theta}{\sim}(F,K)=(F,K)^r$.

Proof: Let $\emptyset_K$=(S,K), where for all $\omega\in$K, S(ω)=$\emptyset$. $(F,K)\underset{\theta}{\sim}$(S,K)=(H,K), where for all $\omega\in$K,

$$H(\omega)=\begin{cases} F(\omega), & \omega\in K-K=\emptyset \\ F'(\omega)\cap S'(\omega), & \omega\in K\cap K=K \end{cases}$$

Hence, for all $\omega\in$K, H(ω)=F'(ω)∩S'(ω)=F'(ω)∩U= F'(ω). Thus (H,K)= $(F,K)^r$.

7) $(F,K)\underset{\theta}{\sim}\emptyset_E=(F,K)^r$.

Proof: Let $\emptyset_E$=(S,E), where for all $\omega\in$E, S(ω)=$\emptyset$. Let $(F,K)\underset{\theta}{\sim}$(S,E)=(H,K), where for all $\omega\in$K,

$$H(\omega)=\begin{cases} F(\omega), & \omega\in K-E=\emptyset \\ F'(\omega)\cap S'(\omega), & \omega\in K\cap E=K \end{cases}$$

Thus, for all $\omega\in$K, H(ω)=F'(ω)∩S'(ω)=F'(ω)∩U=F'(ω), so (H,K)= $(F,K)^r$.

8) $(F,K)\underset{\theta}{\sim}\emptyset_\emptyset$=(F,K).

Proof: Let $\emptyset_\emptyset$=(S, $\emptyset$) and (F,K)$\underset{\theta}{\sim}$(S, $\emptyset$)=(H,K). Hence, for all $\omega\in$K,

$$H(\omega)=\begin{cases} F(\omega), & \omega\in K-\emptyset=K \\ F'(\omega)\cap S'(\omega), & \omega\in K\cap\emptyset=\emptyset \end{cases}$$

Thus, for all $\omega\in$K, H(ω)=F(ω), (H,K)=(F,K). That is, $\emptyset_\emptyset$ is the right identity element for the operation $\underset{\theta}{\sim}$ in $S_E(U)$.

9) $\emptyset_\emptyset\underset{\theta}{\sim}(F,K)=\emptyset_\emptyset$.

Proof: Let $\emptyset_\emptyset$=(S, $\emptyset$) and (S,$\emptyset$)$\underset{\theta}{\sim}$(F,K)=(H,$\emptyset$). Since $\emptyset_\emptyset$ is the only soft set whose parameter set is the empty set, (H,$\emptyset$)= $\emptyset_\emptyset$.

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That is, in $S_E(U)$, for the operation $\tilde{\theta}$, the left inverse of each element with respect to the right identity element $\emptyset_\emptyset$ is the soft set $\emptyset_\emptyset$. Moreover, in $S_E(U)$, the left absorbing element of the $\tilde{\theta}$ operation is the soft set $\emptyset_\emptyset$.

Theorem 3.3.2. From the properties of (1), (2), (8), and (9), the algebraic structure ($S_E(U)$, $\tilde{\theta}$) is a right-left system with the right identity $\emptyset_\emptyset$ and the left inverses of each element is $\emptyset_\emptyset$ under the condition $K \cap Y \cap D = K \cap Y' \cap D = \emptyset$, where (F,K), (G,Y), and (H, D) are the elements of $S_E(U)$.

10) $(F,K)\tilde{\theta}\, U_K = U_K\, \tilde{\theta}(F,K) = \emptyset_K$.

Proof: Let $U_K=(T,K)$, where for all $\omega \in K$, $T(\omega)=U$. Let $(F,K)\tilde{\theta}(T,K)=(H,K)$, where for all $\omega \in K$,

$$H(\omega)=\begin{cases} F(\omega), & \omega \in K-K=\emptyset \\ F'(\omega)\cap T'(\omega), & \omega \in K\cap K=K \end{cases}$$

Thus, for all $\omega \in K$, $H(\omega)=F'(\omega)\cap T'(\omega)=F'(\omega)\cap\emptyset=\emptyset$ so, $(H,K)=\emptyset_K$.

11) $(F,K)\tilde{\theta}\, U_E=\emptyset_K$.

Proof: Let $U_E=(T,E)$, where for all $\omega \in E$, $T(\omega)=U$. Let $(F,K)\tilde{\theta}(T,E)=(H,K)$, where for all $\omega \in K$,

$$H(\omega)=\begin{cases} F(\omega), & \omega \in K-E=\emptyset \\ F'(\omega)\cap T'(\omega), & \omega \in K\cap E=K \end{cases}$$

Thus, for all $\omega \in K$, $H(\omega)=F'(\omega)\cap T'(\omega)=F'(\omega)\cap\emptyset=\emptyset$. Hence, $(H,K)=\emptyset_K$.

12) $(F,K)\tilde{\theta}(F,K)^r=(F,K)^r\tilde{\theta}(F,K) = \emptyset_K$.

Proof: Let $(F,K)^r=(H,K)$, where for all $\omega \in K$, $H(\omega)=F'(\omega)$. Let $(F,K)\tilde{\theta}(H,K)=(T,K)$, where for all $\omega \in K$,

$$T(\omega)=\begin{cases} F(\omega), & \omega\in K-K=\emptyset \\ F'(\omega)\cap H'(\omega), & \omega\in K\cap K=K \end{cases}$$

Thus, for all $\omega\in K$, $T(\omega)=F'(\omega)\cap H'(\omega)=F'(\omega)\cap F(\omega)=\emptyset$. Thus, $(T,K)=\emptyset_K$.

13) $[(F,K)\underset{\theta}{\tilde{}}(G,Y)]^r=(F,K)\underset{\cup}{\overset{*}{\sim}}(G,Y)$.

Proof: Let $(F,K)\underset{\theta}{\tilde{}}(G,Y)=(H,K)$, where for all $\omega\in K$,

$$H(\omega)=\begin{cases} F(\omega), & \omega\in K-Y \\ F'(\omega)\cap G'(\omega), & \omega\in K\cap Y \end{cases}$$

Let $(H,K)^r=(T,K)$, where for all $\omega\in K$,

$$T(\omega)=\begin{cases} F'(\omega), & \omega\in K-Y \\ F(\omega)\cup G(\omega), & \omega\in K\cap Y \end{cases}$$

Hence, $(T,K)=(F,K)\underset{\cup}{\overset{*}{\sim}}(G,Y)$.

14) $(F,K)\underset{\theta}{\tilde{}}(G,K)=U_K \Leftrightarrow (F,K)=(G,K)=\emptyset_K$.

Proof: Let $(F,K)\underset{\theta}{\tilde{}}(G,K)=(T,K)$, where for all $\omega\in K$,

$$T(\omega)=\begin{cases} F(\omega), & \omega\in K-K=\emptyset \\ F'(\omega)\cap G'(\omega), & \omega\in K\cap K=K \end{cases}$$

Since $(T,K)=\emptyset_K$, for all $\omega\in K$, $T(\omega)=\emptyset$. Thus, for all $\omega\in K$, $T(\omega)=F'(\omega)\cap G'(\omega)=U \Leftrightarrow$ for all $\omega\in K$, $F'(\omega)=U$ and $G'(\omega)=U$ $\Leftrightarrow$ for all $\omega\in K$, $F(\omega)=\emptyset$ and $G(\omega)=\emptyset \Leftrightarrow (F,K)=(G,K)=\emptyset_K$.

15) $\emptyset_K \cong\!\!\!\subseteq (F,K)\underset{\theta}{\tilde{}}(G,Y)$ and $\emptyset_Y \cong\!\!\!\subseteq (G,Y)\underset{\theta}{\tilde{}}(F,K)$.

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16) $(F,K)\tilde{\theta}(G,Y) \tilde{\subseteq} U_K$ and $(G,Y)\tilde{\theta}(F,K) \tilde{\subseteq} U_Y$.

17) $(F,K)\tilde{\theta}(G,K) \tilde{\subseteq} (F,K)^r$ and $(F,K)\tilde{\theta}(G,K) \tilde{\subseteq} (G,K)^r$.

Proof: Let $(F,K)\tilde{\theta}(G,K)=(H,K)$. For all $\omega\in K$,

$$H(\omega)=\begin{cases} F(\omega), & \omega\in K-K=\emptyset \\ F'(\omega)\cap G'(\omega), & \omega\in K\cap K=K \end{cases}$$

Since for all $\omega\in K$, $H(\omega)=F'(\omega)\cap G'(\omega)\subseteq F'(\omega)$, $(F,K)\tilde{\theta}(G,K)\tilde{\subseteq}(F,K)^r$. $(F,K)\tilde{\theta}(G,K)\tilde{\subseteq}(G,K)^r$ can be shown similarly.

18) If $(F,K)\tilde{\subseteq}(G,K)$, then $(F,K)\tilde{\theta}(G,K)=(G,K)^r$.

Proof: Let $(F,K)\tilde{\subseteq}(G,K)$. Then for $\omega\in K$, $F(\omega)\tilde{\subseteq} G(\omega)$. Let $(F,K)\tilde{\theta}(G,K)=(H,K)$, where for all $\omega\in K$,

$$H(\omega)=\begin{cases} F(\omega), & \omega\in K-K=\emptyset \\ F'(\omega)\cap G'(\omega), & \omega\in K\cap K=K \end{cases}$$

Since for all $\omega\in K$, if $F(\omega)\tilde{\subseteq} G(\omega)$, then $G'(\omega)\tilde{\subseteq} F'(\omega)$. Hence, for all $\omega\in K$, $H(\omega)=F'(\omega)\cap G'(\omega)=G'(\omega)$. Thus, $(F,K)\tilde{\theta}(G,Y)=(G,K)^r$.

19) If $(F,K)\tilde{\subseteq}(G,K)$, then $(H,Z)\tilde{\theta}(G,K) \tilde{\subseteq} (H,Z)\tilde{\theta}(F,K)$ and $(G,K)\tilde{\theta}(H,K) \tilde{\subseteq} (F,K)\tilde{\theta}(H,K)$.

Proof: Let $(F,K)\tilde{\subseteq}(G,K)$. Thus, for all $\omega\in K$, $F(\omega)\subseteq G(\omega)$ so, for all $\omega\in K$, $G'(\omega)\subseteq F'(\omega)$. Let $(H,Z)\tilde{\theta}(G,K)=(W,Z)$. Hence, for all $\omega\in Z$,

$$W(\omega)=\begin{cases} H(\omega), & \omega\in Z-K \\ H'(\omega)\cap G'(\omega), & \omega\in Z\cap K \end{cases}$$

Let $(H,Z)\tilde{\theta}(F,K)=(L,Z)$, where for all $\omega\in Z$,

$$L(\omega)=\begin{cases} H(\omega), & \omega\in Z-K \\ H'(\omega)\cap F'(\omega), & \omega\in Z\cap K \end{cases}$$

Hence, for all $\omega\in Z-K$, $W(\omega)=H(\omega)\subseteq H(\omega)=L(\omega)$, and for all $\omega\in Z\cap K$ $W(\omega)=H'(\omega)\cap G'(\omega)\subseteq H'(\omega)\cap F'(\omega)=L(\omega)$. Thus, $(H,Z)\tilde{\theta}(G,K)\,\tilde{\subseteq}(H,Z)\tilde{\theta}(F,K)$. Moreover, for all $\omega\in K$, since $G'(\omega)\cap H'(\omega)\subseteq F'(\omega)\cap H'(\omega)$, $(G,K)\,\tilde{\theta}(H,K)\,\tilde{\subseteq}(F,K)\,\tilde{\theta}(H,K)$.

20) If $(H,Z)\tilde{\theta}(G,K)\,\tilde{\subseteq}(H,Z)\tilde{\theta}(F,K)$, then $(F,K)\tilde{\subseteq}(G,K)$ needs not be true. Similarly, if $(G,K)\,\tilde{\theta}(H,K)\,\tilde{\subseteq}\,(F,K)\,\tilde{\theta}(H,K)$, then $(F,K)\,\tilde{\subseteq}\,(G,K)$ needs not be true. That is, the converse of Theorem 3.3. (19) is not true.

Proof: To demonstrate that the converse of Theorem 3.3. (19) is not true, let's provide an example. Let $E=\{e_1,e_2,e_3,e_4,e_5\}$ be the parameter set, $K=\{e_1,e_3\}$ and $Z=\{e_1,e_3,e_5\}$ be two subsets of E, $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set. Let (F,K), (G,K), and (H,Z) be soft sets over U as follows: $(F,K)=\{(e_1,\{h_2,h_5\}),(e_3,\{h_1,h_2,h_5\})\}$, $(G,K)=\{(e_1,\{h_2\}),(e_3,\{h_1,h_2\})\}$, $(H,Z)=\{(e_1,U),(e_3,U),(e_5,\{h_2,h_5\})\}$. Let $(H,Z)\tilde{\theta}(G,K)=(L,Z)$, where for all $\omega\in Z-K=\{e_5\}$, $L(e_5)=H(e_5)=\{h_2,h_5\}$, *for all* $\omega\in Z\cap K=\{e_1,e_3\}$, $L(e_1)=H'(e_1)\cap G'(e_1)=\emptyset$, $L(e_3)=H'(e_3)\cap G'(e_3)=\emptyset$. Thus, $(H,Z)\tilde{\theta}(G,K)=\{(e_1,\emptyset),(e_3,\emptyset),(e_5,\{h_2,h_5\})\}$. Now let $(H,Z)\,\tilde{\theta}(F,K)=(W,Z)$. Then, $W(e_5)=H(e_5)=\{h_2,h_5\}$, $W(e_1)=H'(e_1)\cap F'(e_1)=\emptyset$, $W(e_3)=H'(e_3)\cap F'(e_3)=\emptyset$. Thus, $(H,Z)\tilde{\theta}(F,K)=\{(e_1,\emptyset),(e_3,\emptyset),(e_5,\{h_2,h_5\})\}$. Thus, $(H,Z)\tilde{\theta}(G,K)\,\tilde{\subseteq}\,(H,Z)\tilde{\theta}(F,K)$, however, (F,K) is not a soft subset of (G,K). Similarly, if $(G,K)\,\tilde{\theta}(H,K)\,\tilde{\subseteq}\,(F,K)\,\tilde{\theta}(H,K)$, then $(F,K)\,\tilde{\subseteq}\,(G,K)$ needs not to be true can be shown by taking $(H,K)=\{(e_1,U),(e_3,U)\}$ in the same example.

21) If $(F,T)\,\tilde{\subseteq}\,(G,T)$ and $(K,T)\,\tilde{\subseteq}\,(L,T)$, then $(G,T)\tilde{\theta}\,(L,T)\,\tilde{\subseteq}(F,T)\tilde{\theta}\,(K,T)$.

Proof: Let $(F,T)\,\tilde{\subseteq}\,(G,T)$ ve $(K,T)\tilde{\subseteq}\,(L,T)$. Thus, for all $\omega\in T$, $F(\omega)\subseteq G(\omega)$ and $K(\omega)\subseteq L(\omega)$. Thus, for all $\omega\in T$, $G'(\omega)\subseteq F'(\omega)$ and $L'(\omega)\subseteq K'(\omega)$. Let $(G,T)\tilde{\theta}(L,T)=(M,T)$. Hence, for all $\omega\in T$, $M(\omega)=G'(\omega)\cap L'(\omega)$. Let $(F,T)\tilde{\theta}(K,T)=(N,T)$. Hence, for all $\omega\in T$, $N(\omega)=F'(\omega)\cap K'(\omega)$. Since for all $\omega\in T$, $G'(\omega)\subseteq F'(\omega)$ and $L'(\omega)\subseteq K'(\omega)$, $M(\omega)=G'(\omega)\cap L'(\omega)\subseteq F'(\omega)\cap K'(\omega)=N(\omega)$. Hence, $(G,T)\tilde{\theta}\,(L,T)\,\tilde{\subseteq}(F,T)\tilde{\theta}(K,T)$.

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22) $(F,K)\tilde{\theta}(G,K) \tilde{\subseteq} (F,K)\tilde{*}(G,K)$.

Proof: Let $(F,K)\tilde{\theta}(G,K) = (T,K)$. Thus, for all $\omega \in K$,

$$T(\omega)=\begin{cases} F(\omega), & \omega\in K\text{-}K=\emptyset \\ F'(\omega)\cap G'(\omega), & \omega\in K\cap K=K \end{cases}$$

Let $(F,K)\tilde{*}(G,K) = (W,K)$. Thus, for all $\omega \in K$,

$$W(\omega)=\begin{cases} F(\omega), & \omega\in K\text{-}K=\emptyset \\ F'(\omega)\cup G'(\omega), & \omega\in K\cap K=K \end{cases}$$

Since for all $\omega\in K$, $T(\omega)=F'(\omega)\cap G'(\omega)\subseteq F'(\omega)\cup G'(\omega)=W(\omega)$. Hence, $(F,K)\tilde{\theta}(G,K) \tilde{\subseteq} (F,K)\tilde{*}(G,K)$.

4. Distribution Rules

In this section, the distributions of soft binary piecewise theta operation over other soft set operations are examined in detail, and many interesting algebraic structures are obtained.

Proposition 4.1. Let (F,K), (G,Y), and (H,D) be soft sets over U. Then, the soft binary piecewise theta operation distributes over restricted operations as follows, where $K\cap Y\cap D=\emptyset$.

1) $[(F,K)\cup_R(G,Y)]\tilde{\theta}(H,D)=[(F,K)\tilde{\theta}(H,D)]\cup_R[(G,Y)\tilde{\theta}(H,D)]$.

2) $[(F,K)\cap_R(G,Y)]\tilde{\theta}(H,D)=[(F,K)\tilde{\theta}(H,D)]\cap_R[(G,Y)\tilde{\theta}(H,D)]$.

Corollary 4.1.1. $(S_E(U),\cup_R,\tilde{\theta})$ is an additive commutative and additive idempotent (right) near-semiring without zero and unity under certain conditions.

Corollary 4.1.2. $(S_E(U),\cap_R,\tilde{\theta})$ is an additive commutative and additive idempotent (right) near-semiring without zero and unity under certain conditions.

Proposition 4.2. Let (F,K), (G,Y), and (H,D) be soft sets over U. Then, the distributions of the soft binary piecewise theta operation over extended soft set operations are as follows, where K∩(YΔD)= K∩Y∩D=∅.

1) $(F,K)\tilde{\theta}[(G,Y)\cup_{\varepsilon}(H,D)]=[(F,K)\tilde{\theta}(G,Y)]\cup_{\varepsilon}[(F,K)\tilde{\theta}(H,D)]$.

2) $(F,K)\tilde{\theta}[(G,Y)\cap_{\varepsilon}(H,D)]=[(F,K)\tilde{\theta}(G,Y)]\cap_{\varepsilon}[(F,K)\tilde{\theta}(H,D)]$.

RHS Distributions: The following holds, where K∩Y∩D=∅.

1) $[(F,K)\cap_{\varepsilon}(G,Y)]\tilde{\theta}(H,D)=[(F,K)\tilde{\theta}(H,D)]\cap_{\varepsilon}[(G,Y)\tilde{\theta}(H,D)]$.

2) $[(F,K)\cup_{\varepsilon}(G,Y)]\tilde{\theta}(H,D)=[(F,K)\tilde{\theta}(H,D)]\cup_{\varepsilon}[(G,Y)\tilde{\theta}(H,D)]$.

3) $[(F,K)\setminus_{\varepsilon}(G,Y)]\tilde{\theta}(H,D)=[(F,K)\tilde{\theta}(H,D)]\setminus_{\varepsilon}[(G,Y)\tilde{\theta}(H,D)]$

4) $[(F,K)\Delta_{\varepsilon}(G,Y)]\tilde{\theta}(H,D)=[(F,K)\tilde{\theta}(H,D)]\Delta_{\varepsilon}[(G,Y)\tilde{\theta}(H,D)]$.

5) $[(F,K)+_{\varepsilon}(G,Y)]\tilde{\theta}(H,D)=[(F,K)\tilde{\theta}(H,D)]+_{\varepsilon}[(G,Y)\tilde{\theta}(H,D)]$.

6) $[(F,K)\gamma_{\varepsilon}(G,Y)]\tilde{\theta}(H,D)=[(F,K)\tilde{\theta}(H,D)]\gamma_{\varepsilon}[(G,Y)\tilde{\theta}(H,D)]$.

7) $[(F,K)*_{\varepsilon}(G,Y)]\tilde{\theta}(H,D)=[(F,K)\tilde{\theta}(H,D)]*_{\varepsilon}[(G,Y)\tilde{\theta}(H,D)]$.

8) $[(F,K)\theta_{\varepsilon}(G,Y)]\tilde{\theta}(H,D)=[(F,K)\tilde{\theta}(H,D)]\theta_{\varepsilon}[(G,Y)\tilde{\theta}(H,D)]$.

Corollary 4.2.1. $(S_E(U),\cup_{\varepsilon},\tilde{\theta})$ and $(S_E(U),\cap_{\varepsilon},\tilde{\theta})$ are additive commutative and additive idempotent (right) near-semirings with zero but without unity and zero symettric property under certain conditions. Similarly, $(S_E(U),\setminus_{\varepsilon},\tilde{\theta})$, $(S_E(U),\Delta_{\varepsilon},\tilde{\theta})$, $(S_E(U),+_{\varepsilon},\tilde{\theta})$, $(S_E(U),\gamma_{\varepsilon},\tilde{\theta})$, $(S_E(U),\lambda_{\varepsilon},\tilde{\theta})$, $(S_E(U),*_{\varepsilon},\tilde{\theta})$, $(S_E(U),\theta_{\varepsilon},\tilde{\theta})$ are additive commutative, not idempotent (right) near-semirings with zero but without unity and zero symmetric property under certain conditions.

Corollary 4.2.2. $(S_E(U),\cup_{\varepsilon},\tilde{\theta})$ and $(S_E(U),\cap_{\varepsilon},\tilde{\theta})$ are additive commutative and additive idempotent semirings without zero and unity under certain conditions.

Proposition 4.3. Let (F,K), (G,Y), and (H,D) be soft sets on U. Then, the distribution of the soft binary piecewise theta operation over soft binary piecewise operations are as follows: The following holds where K∩Y∩D=∅.

1) $[(F, K) \,\widetilde{\cap}\, (G,Y)] \,\widetilde{\theta}\, (H,D)=[(F,K) \,\widetilde{\theta}\, (H,D)] \,\widetilde{\cap}\, [(G, Y) \,\widetilde{\theta}\, (H,D)]$.

2) $[(F, K) \,\widetilde{\cup}\, (G,Y)] \,\widetilde{\theta}\, (H,D)=[(F,K) \,\widetilde{\theta}\, (H,D)] \,\widetilde{\cup}\, [(G, Y) \,\widetilde{\theta}\, (H,D)]$.

3) $[(F, K) \,\widetilde{\setminus}\, (G,Y)] \,\widetilde{\theta}\, (H,D)=[(F,K) \,\widetilde{\theta}\, (H,D)] \,\widetilde{\setminus}\, [(G, Y) \,\widetilde{\theta}\, (H,D)]$.

4) $[(F, K) \,\widetilde{\Delta}\, (G,Y)] \,\widetilde{\theta}\, (H,D)=[(F,K) \,\widetilde{\theta}\, (H,D)] \,\widetilde{\Delta}\, [(G, Y) \,\widetilde{\theta}\, (H,D)]$.

5) $[(F, K) \,\widetilde{+}\, (G,Y)] \,\widetilde{\theta}\, (H,D)=[(F,K) \,\widetilde{\theta}\, (H,D)] \,\widetilde{+}\, [(G, Y) \,\widetilde{\theta}\, (H,D)]$.

6) $[(F, K) \,\widetilde{\gamma}\, (G,Y)] \,\widetilde{\theta}\, (H,D)=[(F,K) \,\widetilde{\theta}\, (H,D)] \,\widetilde{\gamma}\, [(G, Y) \,\widetilde{\theta}\, (H,D)]$.

7) $[(F, K) \,\widetilde{*}\, (G,Y)] \,\widetilde{\theta}\, (H,D)=[(F,K) \,\widetilde{\theta}\, (H,D)] \,\widetilde{*}\, [(G, Y) \,\widetilde{\theta}\, (H,D)]$

8) $[(F, K) \,\widetilde{\theta}\, (G,Y)] \,\widetilde{\theta}\, (H,D)=[(F,K) \,\widetilde{\theta}\, (H,D)] \,\widetilde{\theta}\, [(G, Y) \,\widetilde{\theta}\, (H,D)]$.

9) $[(F,K) \,\widetilde{\lambda}\, (G,Y)] \,\widetilde{\theta}\, (H,D)=[(F,K) \,\widetilde{*}\, (H,D)] \,\widetilde{\lambda}\, [(G, Y) \,\widetilde{\theta}\, (H,D)]$.

Corollary 4.3.1. $(S_E(U),\widetilde{\cap},\widetilde{\theta})$ and $(S_E(U),\widetilde{\cup},\widetilde{\theta})$ are additive idempotent, non-commutative (right) near-semirings without zero and unity under certain conditions.

Corollary 4.3.2. $(S_E(U),\widetilde{\setminus},\widetilde{\theta})$, $(S_E(U),\widetilde{\Delta},\widetilde{\theta})$, $(S_E(U),\widetilde{+},\widetilde{\theta})$, $(S_E(U),\widetilde{\gamma},\widetilde{\theta})$, $(S_E(U),\widetilde{*},\widetilde{\theta})$, $(S_E(U),\widetilde{\theta},\widetilde{\theta})$ are all not idempotent and non-commutative (right) near-semirings without zero and unity under the condition $T\cap Z'\cap M = T\cap Z\cap M =\emptyset$, where (F,T), (G,Z) and (H,M) are soft sets over U. Here, note that Yavuz (2024) showed that the first operation is associative in $S_E(U)$ under the condition $T\cap Z'\cap M=T\cap Z\cap M=\emptyset$ (for $\widetilde{\Delta}$, under the condition $T\cap Z'\cap M=\emptyset$)

5. Conclusion

This study introduces a novel sort of soft set operation. We aim to contribute to the field of soft set theory by proposing a novel soft set operation that we term the "soft binary piecewise theta operation" and by carefully studying the algebraic structures related to it and other novel soft set operations in the class of soft sets. In particular, the algebraic properties of this new soft set operation are thoroughly examined, and the distributions of the soft binary piecewise theta operation over various types of soft set operations are studied. Considering the distribution laws and the algebraic properties of the soft set operations, a detailed analysis of the algebraic structures generated by the set of soft sets with these operations is provided. We show that the collection of soft sets over the universe with the soft binary piecewise theta operation, and other kinds of soft sets, form various important algebraic structures, such as semirings and near-semirings. Future research endeavors may thoroughly examine other variants of soft binary piecewise operations, along with the associated characteristics and distributions.

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