Jurnal Matematika Vol. 15, No.1, Juni 2025, pp. 13-22 ISSN: 1693-1394
Article DOI: 10.24843/JMAT.2025.v15.i01.p182

# Geometric Brownian Motion On Prediction Of Foreign Exchange Rate: A Study on Indonesian Rupiah Rate

**Mohammad Dian Purnama**
Mathematics, Faculty of Mathematics and Natural Sciences, State University of Surabaya, Ketintang, Surabaya, East Java, 60231, Indonesia
e-mail: mohammaddian.20053@mhs.unesa.ac.id

**Ilyasa Pahlevi Reza**
Mathematics, Faculty of Military Mathematics and Natural Sciences, Republic of Indonesia Defense University, Sukahati, Bogor, West Java, 16810, Indonesia
e-mail: ilyasapahlevi@gmail.com

**Rizky Sampoerna**
Information Technology, School of Computing, Telkom University, Bojongsoang, Bandung, West Java, 40257, Indonesia
e-mail: rizkysampoerna@student.telkomuniversity.ac.id

***Abstract*****:** *Exchange rates wield significant influence on economic stability and activity, exerting considerable effects on overall economic conditions. A comprehensive understanding of the dynamics governing the Rupiah's exchange rate against foreign currencies is essential for accurate prognostication and strategic decision-making across economic and commercial sectors. Geometric Brownian Motion (GBM) emerges as a prevalent tool employed in predictive modelling. This necessitates the acquisition and analysis of data encompassing exchange rates involving three key currencies relative to the Indonesian Rupiah (USD, EUR, SGD). Utilizing GBM facilitates the projection of future exchange rate movements with a notable degree of reliability. Notably, the mean absolute percentage error (MAPE) values derived from six distinct GBM models underscore a remarkable forecasting precision, consistently falling below the 10% threshold. Furthermore, an extended evaluation using 500 iterations demonstrates significantly smaller MAPE values, indicating a statistically significant improvement in accuracy for predicting exchange rate fluctuations compared to alternative methodologies. These findings offer valuable insights for policymakers, businesses, and investors, empowering them to navigate the complex terrain of global currency markets with heightened confidence and efficacy.*

***Keywords****: Geometric Brownian Motion, Exchange Rate, Rupiah*

## 1. Introduction

The exchange rate represents the price of one unit of foreign currency in domestic currency. This exchange rate between the currencies of two countries or regions is used for current and future transactions (Amalia, et.al. 2022). The currency exchange rate also reflects the price of one unit of currency in other currency units. The determination of this currency exchange rate occurs in the foreign exchange market, which is where the currency trading of various countries occurs. Currency exchange rates are one of the key macroeconomic factors that have a significant impact on economic stability and activity, as changes in exchange rates can affect overall economic conditions (Prasetyo, et.al. 2021).

The value of currencies between one country and another is different. The difference in the value of a country's currency is strongly influenced by capital flows between countries. The instability of exchange rates or the strengthening of other countries' currencies against a country's currency can cause the country's economic conditions to deteriorate (Budiastawa, et.al. 2019). Currency exchange rates also have an important role because they have various impacts, one of which is stock returns which also support a country's economy (Nugroho & Hermuningsih, 2020).

By considering the above description, it is important to take steps that allow in-depth understanding of the Rupiah exchange rate against foreign currencies. These steps can produce accurate predictions, so that concrete actions can be taken after the Rupiah exchange rate against foreign currencies through the prediction process, especially in the economic sector and business activities (Are & Sitorus, 2020).

Many researchers have conducted research on exchange rate prediction using various methods. One of them is the research conducted by Zyad, et.al. (2021). The study has simulated the USD to IDR exchange rate with the Long Short-Term Memory Method (LSTM). There are other studies on forecasting the exchange rate against the dollar and yuan using FTS-markov chain conducted by Amalutfia and Hafiyusholeh (2020). There is a need for other methods to forecast other exchange rates considering that exchange rates greatly affect the economic activities of a country.

Many methods can be used to obtain predictions of foreign exchange rates using various approaches. One of the approaches in prediction is Geometric Brownian Motion (GBM). GBM is one of the mathematical models used to predict time series data such as exchange rates, stock prices and other financial instruments. The GBM model is also a continuous-time stochastic model, wherein its random variables follow Brownian Motion. The model assumes that the price of a stock or other financial asset will fluctuate randomly in the short term but tend to rise or fall in the long term. (Bhakti, 2022). Therefore, this research will use the GBM method in predicting the exchange rate.

## 2. Methodology

### 2.1 *Data*

This research uses rupiah exchange rate data from three leading currencies in the world, each of which has specific characteristics for each continent. This data will be the main research material, which is then processed and validated to support the prediction data generated. The data used are three currency exchange rates against the Indonesian Rupiah (IDR), namely the American Dollar (USD), Euro (EUR), and Singapore Dollar (SGD). Sourced from the Yahoo Finance, with the division between training data and testing data is as follows. Training data is taken from the period January 1, 2024 to February 12, 2024, while testing data is taken from the period February 13, 2024 to February 29, 2024. The data will be organized in tabular form for presentation.

Table 1. Data

| No | ***Currency*** | ***Total Data*** | |
|---|---|---|---|
| | | ***Training*** | ***Testing*** |
| 1 | USD to IDR | 31 | 13 |
| 2 | EUR to IDR | 31 | 13 |
| 3 | SGD to IDR | 31 | 13 |

The research method of forecasting exchange rates using the GBM method is illustrated in Figure 1.

The procedures of data analysis conducted in this study are outlined as follows:

1. Calculation of return value
   The formula for calculating stock price returns is (Kholiliah & Safitri, 2015)

$$R_{it} = \frac{P_{it}-P_{it-1}}{P_{it-1}} \quad ,(1)$$

   Where $R_{it}$ is the return of stock $i$ on day $t$, $P_{it}$ is the price of stock $i$ on day $t$, and $P_{it-1}$ is the price of stock $i$ on day $t-1$.
2. Normality testing of data
   The normality test aims to test whether the dependent and independent variables have a normal distribution in the regression model and whether the residual values have a normal distribution. A good regression model has typical or close-to-normal residual values. The normality test used in this research is Shapiro Wilk, namely

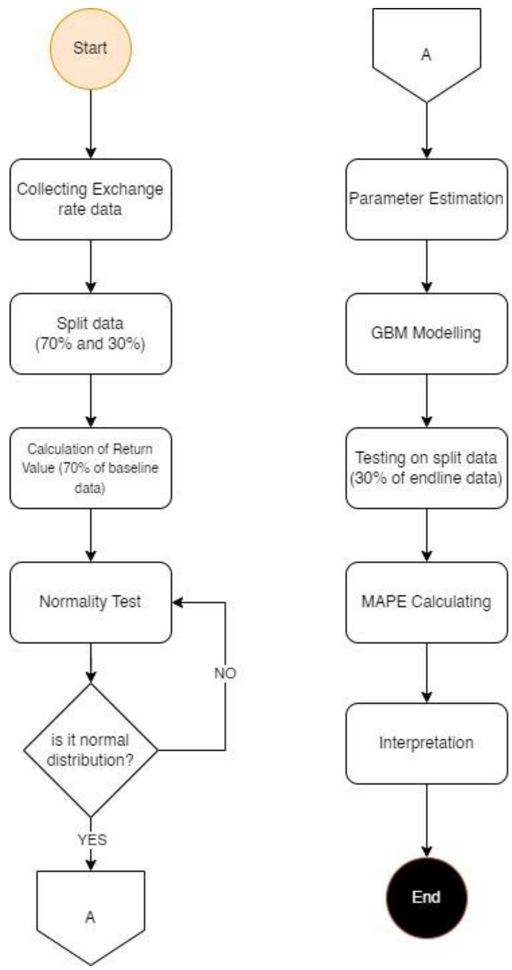


Figure 1. Research Flowchart

with the criteria that if significant Shapiro Wilk P-Value less than 0.05 then the data is not normal, conversely if significant Shapiro Wilk P-Value more than 0.05, then the data is normal (Sugiarto, 2023).

3. Calculation of estimation parameters
    a. Volatility
       Volatility measures changes in a security or market index's price over time. The higher the stock volatility, the more frequently the stock fluctuates, and vice versa if the value is low. It shows that the security price is relatively stable.

$$\sigma = \frac{s}{\sqrt{t}}, \tag{2}$$

       Where $\sigma$ is volatility, $s$ is standard deviation, and $t$ is the total of time.
    b. Standard Deviation

In the volatility formula, there is a standard deviation which can be calculated with the following formula:

$$s = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(R_k - \bar{R})^2}, \tag{3}$$

Where $s$ is the standard deviation, $n$ is the sum of return total, $R_k$ is the return value of $k$, and $\bar{R}$ is the average of return.

c. Drift

Drift is a term used to describe a stock's expected annualized rate of return. To find the stock drift as follows (Azizah, et.al., 2020):

$$\mu = \frac{\bar{R}}{t} + \frac{\sigma^2}{2}, \tag{4}$$

Where $\mu$ is drift, and $\sigma^2$ is volatility squared.

4. Modelling with Geometric Brownian Motion (GBM)

Geometric Brownian Motion (GBM) is a stochastic process that models stock price movements. GBM is the result of a combination of Wiener and exponential processes, so it can be used to model stock price movements with characteristics as a stochastic process that follows a normal distribution and has an exponential upward trend (Liden, 2018). GBM process is used in stock price prediction by assuming that the movement of stock prices over a certain period is a random process described by equation (Hull, 2021):

$$dS = \mu S dt + \sigma S dw, \tag{5}$$

Where $dS$ is a stock price change, $dt$ is the time interval between observations, and $dW$ is the change in the Wiener process. The above model can also be written as follows:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t), \tag{6}$$

If the equation is written in terms of long-time intervals between consecutive values, the following equation is obtained:

$$\frac{dS(t)}{s(t)} = d\big(lnS(t)\big) = lnS(t) - lnS(t-1) = \ln\left(\frac{s(t)}{s(t-1)}\right), \tag{7}$$

$$\ln\left(\frac{s(t)}{s(t-1)}\right) = \mu dt = \sigma dW(t), \tag{8}$$

The above equation is obtained from the application formula, which will be explained in general. For each function $G(S,t)$ from $S$ and $t$ where $X$ satisfies the following stochastic differential equation:

$$dX = adt + bdW(t), \tag{9}$$

For some constants, $a$, $b$, and $dW(t)$ are the Brownian motions. Then, the Ito formula itself is defined as follows:

$$dG = \left(\frac{\partial G}{\partial s}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial s^2}b^2\right)dt + \frac{\partial G}{\partial s}dW, \quad (10)$$

Then, to determine $G(t,S) = ln(S(t))$ in order to fulfil the GBM form, the equation is derived and entered into the Ito formula obtained:

$$d\big(\ln(S(t))\big) = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dW(t), \quad (12)$$

$$\frac{s(t)}{s(t-1)} = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dW(t), \quad (13)$$

If simplified, we will get a stock price estimation model in GBM as follows:

$$S_{t+1} = S_t . e^{\left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma Wt}, \quad (14)$$

5. Calculation of MAPE value on GBM models

   The research employs the Mean Absolute Percentage Error (MAPE) as a metric for model accuracy assessment. MAPE is a commonly utilized method for evaluating estimates, taking into account the impact of the actual values (Azizah, et.al, 2020). The MAPE formula can be calculated as follows:

$$MAPE = \frac{\Sigma\left|\frac{Y_t - F_t}{Y_t}\right|}{n} \times 100\%, \quad (15)$$

   Where $Y_t$ is the value of testing data at time $t$, $F_t$ is the value of estimation data at time $t$, and $n$ is the total of testing data. The smaller the MAPE value, the more accurate the model (Abidin, et.al, 2014). The accuracy assessment scale of MAPE can be seen in the following Table 2.

Table 2. MAPE

| MAPE Value | Description |
|---|---|
| <10% | High Accuracy |
| 11% - 20% | Good Accuracy |
| 21% - 50% | Accuracy within Reasonable Boundaries |
| >51% | Inaccurate Accuracy |

## 3. Result

To simplify estimation, it's essential to compute returns on the data due to the erratic nature of foreign exchange rate data, which can undergo abrupt rises and falls.

Table 3. Data of Return Currency (January 1, 2024 to February 12, 2024)

| The t-th Return | USD/IDR | EUR/IDR | SGD/IDR |
|---|---|---|---|
| $R_1 = \frac{P_1 - P_0}{P_0}$ | 0.0000 | 0.0000 | 0.0001 |
| $R_2 = \frac{P_2 - P_1}{P_1}$ | 0.0075 | -0.0070 | 0.0029 |
| ⋮ | ⋮ | ⋮ | ⋮ |
| $R_{30} = \frac{P_{31} - P_{30}}{P_{30}}$ | -0.0040 | -0.0121 | -0.0030 |

This normality test is used to identify whether random variables follow a normal distribution. This test uses the Shapiro wilk test. The following are the results of the normality test on foreign exchange returns:

Table 4. Normality Test

| No | ***Name of Currency*** | ***P-Value*** |
|---|---|---|
| 1 | USD/IDR | 0.8403 |
| 2 | EUR/IDR | 0.1356 |
| 3 | SGD/IDR | 0.3642 |

From the above results, it can be concluded that the p-value in the normality test, all three, is more significant than α, which is 0.05, so the data can continue to be processed because it is typically distributed.

Before the stock estimation modelling stage, it is necessary to calculate parameter estimates, drift and volatility in the GBM model. Based on the drift and volatility formulas in the previous discussion, the results of the parameter estimation value of the return are as follows:

Table 5. Estimation of Parameter

| No | ***Name of Currency*** | $\sigma$ | $\mu$ |
|---|---|---|---|
| 1 | USD/IDR | 0.0031 | 0.0027 |
| 2 | EUR/IDR | 0.0092 | -0.0029 |
| 3 | SGD/IDR | 0.0030 | -0.0009 |

Based on the above calculations, the estimated value of the volatility and drift parameters of the return of each stock is obtained. After knowing the parameter estimates, the value will be modelled for the value of shares for the next period. From the general equation of the GBM model, the following model is obtained:

Table 6. GBM Model

| No | ***Name of Currency*** | **GBM Model** $(S_{t+1})$ |
|---|---|---|
| 1 | USD/IDR | $15579.50 \cdot e^{\left(0.0027-\frac{1}{2}0.0031^2\right)dt+0.0031Wt}$ |
| 2 | EUR/IDR | $16782.00 \cdot e^{\left(-0.0029-\frac{1}{2}0.0092^2\right)dt+0.0092Wt}$ |
| 3 | SGD/IDR | $11583.62 \cdot e^{\left(-0.0009-\frac{1}{2}0.0030^2\right)dt+0.0030Wt}$ |

This research uses six kinds of iterations 10, 100, 500, 1000, 5000, and 10000. Here many plot of the results of the three predicted currencies with GBM compared to the accurate rates with 10 iterations.

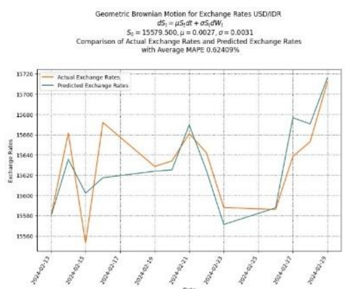


Figure 2. Plot of Comparison Actual and Predicted Exchange Rates for USD/IDR

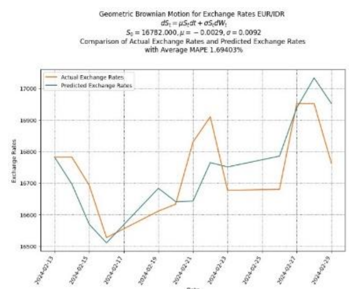


Figure 3. Plot of Comparison Actual and Predicted Exchange Rates for EUR/IDR

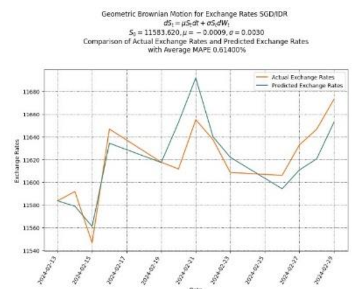


Figure 4. Plot of Comparison Actual and Predicted Exchange Rates for SGD/IDR

The choice of 10 iterations is due to the smallest average MAPE result among all iterations run. Here are the results of the MAPE comparison of the six iterations run.

Table 7. Average MAPE of GBM Model

| Iteration | USD/IDR | EUR/IDR | SGD/IDR |
|---|---|---|---|
| 10 | 0.68% | 1.85% | 0.67% |
| 100 | 0.64% | 1.72% | 0.67% |
| 500 | 0.62% | 1.69% | 0.61% |
| 1000 | 0.64% | 1.76% | 0.65% |
| 5000 | 0.64% | 1.77% | 0.64% |
| 10000 | 0.64% | 1.78% | 0.63% |

Based on the two tables above, the average value of the Mean Absolute Percentage Error (MAPE) of GBM with six iterations has accurate accuracy as indicated by the high level of forecasting accuracy, which is <10%. Of the six iterations run, the results show that the third iteration size of 500 iterations has a MAPE value with factual data as the measurement data smaller than the other iterations.

## 4. Conclusion

Based on the simulation results and discussion above, several conclusions can be drawn regarding stock price predictions using Geometric Brownian Motion, namely the GBM model has been proven to show high forecasting accuracy with an average Mean Absolute Percentage Error (MAPE) value of less than 10%. From the six iterations carried out, the result was that the low iteration had a MAPE value with factual data as measurement data that the third iteration size of 500 iterations has a MAPE value with factual data as the measurement data smaller than the other iterations. Other modifications are needed to reduce MAPE to close to 0 so that the resulting model is more accurate.

## References

Abi Nugroho, G., and Hermuningsih, S. 2020. Pengaruh kurs Rupiah, inflasi dan suku bunga terhadap return saham perusahaan sektor jasa sub konstruksi dan bangunan pada Bursa Efek Indonesia. DERIVATIF: Jurnal Manajemen, 14(1).

Abidin, 2014. Forecasting Share Prices of Small Size Companies in Bursa Malaysia using Geometric Brownian Motion. Applied Mathematics & Information Sciences, 8 (1).

Amalia, S. J., 2022. Perbandingan Metode Moving Average dan Exponential Smoothing pada Peramalan Nilai Tukar Rupiah terhadap Dollar AS. JURIKOM (Jurnal Riset Komputer), 9(4), 974-979.

Are, G. B., and Sitorus, S. H. 2020. Prediksi Nilai Tukar Mata Uang Rupiah Terhadap Dolar Amerika Menggunakan Metode Hidden Markov Model. Coding Jurnal Komputer dan Aplikasi, 8(1).

Bhakti, H. D. 2022. Prediksi Harga Saham Subsektor Farmasi Menggunakan Geometric Brownian Motion. Jurnal Media Informatika Budidarma, 6(1), 395-403.

Budiastawa, I. D. G., Santiyasa, I., and Pramartha, C. R. A. 2019. Prediksi dan akurasi nilai tukar mata uang Rupiah terhadap US Dolar menggunakan radial basis function neural network, Jurnal Elektronik Ilmu Komputer Udayana p-ISSN, 2301, 5373.

J. Liden. 2018. Stock Price Predictions using a Geometric Brownian Motion. Uppsala: Department of Mathematics Uppsala University.

K. Kholilah. and E. Safitri., 2015, Perbedaan Return Saham Sebelum dan Sesudah Pengumuman Dividen pada Perusahaan yang Terdaftar di Jakarta Islamic Index (JII). Fordema, vol. 12, no.1, pp. 71-84.

M. Azizah., M. I. Irawan., and E. R. M. Putri. 2020. Comparison of Stock Price Prediction using Geometric Brownian Motion and Multilayer Perceptron," AIP Conference Proceedings, vol. 2242, no. 1.

Prasetyo, V. R., et al. 2021. Penerapan Aplikasi RapidMiner Untuk Prediksi Nilai Tukar Rupiah Terhadap US Dollar Dengan Metode Regresi Linier. Jurnal Nasional Teknologi dan Sistem Informasi (TEKNOSI), 7(1), 8-17.