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A New Type of Extended Soft Set Operation: Complementary Extended Intersection Operation

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***Abstract*: *Soft set theory is seen as an effective mathematical tool in solving problems involving uncertainty, and has been applied in many theoretical and practical areas since its introduction. The basic concept of the theory is soft set operations. In this context, in this paper, a new kind of soft set operation called complementary extended soft set operation is defined to contribute to the theory. The properties of the operation are examined in detail together with its distributions over other soft set operations to obtain the relationship between complementary extended intersection operation and the others. We demonstrate that the collection of soft sets over with a fixed parameter set, along with the complementary extended intersection operation and other certain types of soft sets, form many well-known and important algebraic structures in classical algebra, including semiring, hemiring, Boolean ring, Boolean Algebra, De Morgan Algebra, Kleene Algebra, and Stone Algebra.*

Keywords: *Soft sets, Complementary extended intersection operation, semiring, hemiring, Algebras*

1. Introduction

People have searched for different solutions over time to solve complex problems involving uncertainty that they encounter in their daily lives. However, existing methods have differed in solving new complex problems that arise under changing conditions. Among the theories put forward to cope with uncertain situations, the theory of fuzzy sets proposed by Zadeh is the most well-known one. Fuzzy set is defined through the membership function. In the rapidly developing fuzzy set theory, some structural problems emerged. Molodtsov (1999) proposed soft set theory far from these structural problems.

Since the soft set was introduced, it has found a place in many theoretical and practical fields and many new studies have been published in the literature. Maji et al. (2003) paved the way for new studies in soft set theory by defining the equality of two soft sets, subset and superset of a soft set, complement of a soft set, soft binary operations such as and/or product, and union and intersection operations for soft sets. Pei and Miao (2005) redefined the concepts of soft subset and intersection of two soft sets based on set theoretical concepts. Then, Ali et al. (2009) proposed some new soft set operations and Sezgin and Atagün (2011) and Ali et al. (2011) analyzed these soft set operations in detail. Sezgin et al. (2019) and Stojanovic (2021) defined extended difference and extended symmetric difference of soft sets and studied their properties in detail in relation to other soft set operations, respectively.

When the studies conducted so far are analyzed, it is seen that soft set operations generally proceed under two main headings as restricted soft set operations and extended soft set operations. Eren and Çalışıcı (2019) defined the soft binary piecewise difference operation for soft sets and examined its properties, and Sezgin and Çalışıcı (2024) studied the properties of this operation in detail. Çağman (2021) proposed the definitions of inclusive complement and exclusive complement of sets as a new concept of set theory, and applied these concepts to group theory. Sezgin et al. (2023a) introduced five new binary complement concepts similar to the binary complement operations in Çağman (2021). Inspired by the new set operations defined in this study, Aybek (2024) proposed many new restricted and extended soft set operations and analyzed their properties. Moreover, the form of soft binary piecewise operation, the pioneer of which is Eren and Çağman (2019), was slightly modified by taking the complement of the image set in the first row, and thus the complementary soft binary piecewise operations have been studied in detail by various researchers (Sezgin and Aybek, 2023, Sezgin and Demirci, 2023; Sezgin and Sarıalioğlu, 2024; Sezgin and Yavuz, 2023a; Sezgin, Aybek, Sezgin and Atagün 2023; Sezgin et al. 2023b). Akbulut (2024) and Demirci (2024), on the other hand, modified the form of the existing extended soft set operations in the literature by taking the complement of the image set in the first and second rows and defined the complementary extended difference, lambda and union, plus and theta respectively, and gave their algebraic properties and relations with other soft set operations. For other applications of soft sets as regards algebraic structures, we refer to the followings: (Çağman et al., 2012; Sezer, 2014; Muştuoğlu et al., 2015; Sezer et al., 2015b; Sezgin et al., 2017; Atagün and Sezgin, 2018; Sezgin, 2018; Mahmood et al., 2018; Jana et al., 2019; Özlü and Sezgin, 2020; Sezgin et al., 2022).

Any specified operation(s) on a set together with that set form(s) an algebraic structure (mathematical structure/mathematical system). Examining the characteristics of the operation defined on a set is one of the most crucial mathematical problems in the framework of algebra, which aims to categorize algebraic structures. In the framework of soft sets as algebraic structures, two main types of soft set collections are examined: A class of soft sets with a fixed set of parameters is represented by the first, and a class of soft sets with variable parameter sets is represented by the second. The ways in which these two types of collections behave differ based on the extra steps taken. Just as fundamental classical set theory operations are essential for soft sets, so too are notions related to soft set operations.

In this paper, to contribute to the theory of soft sets, a new form of soft set operation called complementary extended intersection, is introduced and its properties are discussed in detail. The distribution of complementary extended intersection operation over other types of soft set operations are analysed to obtain the relation of the operation with other soft set operations, and it is shown that the complementary extended intersection operation on a set of soft sets with fixed parameters form many algebraic structures that are well known and highly important in classical algebra such as semiring, hemiring, Boolean ring, Boolean Algebra, De Morgan Algebra, Kleene Algebra, and Stone Algebra with other certain types of soft set operations.

2. Preliminaries

2.1. Definition Let U be the universal set, E be the parameter set, P(U) be the power set of U, and let $D \subseteq E$. A pair (F, D) is called a soft set on U. Here, F is a function given by $F: D \to P(U)$ (Molodtsov, 1999)

The definition of soft set, introduced by Molodtsov, was modified by Çağman and Enginoğlu (2010). Throughout this study, we use the definition of soft set proposed by Molodtsov (1999). The set of all soft sets over U is denoted by $S_E(U)$. Let K be a fixed subset of E, then the set of all soft sets over U with the fixed parameter set K is denoted by $S_K(U)$.

2.2. Definition Let (F,D) be a soft set over U. If $F(\aleph)=\emptyset$ for all $\aleph\in D$, then the soft set (F,D) is called a null soft set with respect to D, denoted by $\emptyset_D$. Similarly, let (F,E) be a soft set over U. If $F(e)=\emptyset$ for all $\aleph\in E$, then the soft set (F,E) is called a null soft set with respect to E, denoted by $\emptyset_E$ (Ali et al., 2011).

A soft set can be defined as $F: \emptyset \longrightarrow P(U)$, where U is a universal set. Such a soft set is called a null soft set and is denoted as $\emptyset_\emptyset$. Thus, $\emptyset_\emptyset$ is the only soft set with an empty parameter set (Ali et al., 2009).

2.3. Definition Let (F,D) be a soft set over U. If $F(\aleph)=U$ for all $\aleph\in D$, then the soft set (F,D) is called a whole soft set with respect to D, denoted by U_D. Similarly, let (F,E) be a soft set over U. If F(e)=U for all ℵ∈E, then the soft set (F,D) is called a whole soft set with respect to E, denoted by U_E (Ali et al., 2009)

2.4. Definition Let (F,D) and (G,Y) be soft sets over U. If D⊆Y and F(ℵ) ⊆G(ℵ) for all ℵ∈D, then (F,D) is said to be a soft subset of (G,Y), denoted by (F,D)$\tilde{\subseteq}$(G,Y). If (G,Y) is a soft subset of (F,D), then (F,D) is said to be a soft superset of (G,Y), denoted by (F,D)$\tilde{\supseteq}$(G,Y). If (F,D)$\tilde{\subseteq}$(G,Y) and (G,Y)$\tilde{\subseteq}$(F,D), then (F,D) and (G,Y) are called soft equal sets (Pei and Maio, 2005)

2.5. Definition Let (F,D) be a soft set over U. The soft complement of (F,), denoted by $(F,D)^r$ =(F^r,D), is defined as follows: for all ℵ∈D, F^r(ℵ)=U-F(ℵ) (Ali et al., 2009)

For more about inclusive complement and exclusive complement, we refer to Çağman (2021) and Sezgin et al. (2023c); about restricted soft set operations, to Ali et al., (2009), Sezgin and Atagün (2011), and Aybek (2024); about extended soft set operations to Maji et al. (2003), Ali et al. (2009), Sezgin et al. (2019), Stojanavic (2021), and Aybek (2024); for more about complementary extended soft set operations to Akbulut (2024), Sarıalioğlu (2024), and Demirci (2024); about soft binary piecewise operations to Eren and Çalışıcı (2019), Sezgin and Yavuz (2023b), Sezgin and Çalışıcı (2024), and Yavuz (2024); about comlementary soft binary piecewise operations to Sezgin and Demirci (2023), Sezgin and Aybek (2023),Sezgin et al. (2023a,2023b), Sezgin and Atagün (2023), Sezgin and Yavuz (2023a), Sezgin and Dagtoros (2023), Sezgin and Çağman (2024), Sezgin and Sarıalioğlu (2024), Sezgin and Sarıalioğlu (2024); about band, semilattice, a bounded semilattice, to Clifford (1954); semiring and hemiring Vandiver (1934); lattice, Boolean algebra, De Morgan algebra, and Stone algebra to Birkhoff (1967); about MV-algebra to Chang (1959). Regarding graph applications and network analysis concerning possible soft set applications, we refer to Pant et al. (2024).

3. Complementary Extended Intersectıon Operatıon

In this section, a new soft set operation called complementary extended intersection operation of soft sets is introduced with its example, and its full algebraic properties are analyzed by comparing it with the intersection operation in classical set theory. Besides, its distribution rules are studied and many algebraic structures are obtained in the collection of soft sets with a fixed parameter set with complementary extended soft set operations and other types of soft set operations.

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3.1.Definition Let (F,Z), (G,C) be soft sets over U.The complementary extended intersection operation ($\cap$) operation of (F,Z) and (G,C) is the soft set (H,K), denoted by $(F,Z)\overset{*}{\cap}_{\varepsilon}(G,C)=(H,K)$, where for all $\aleph\in K=Z\cup C$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

3.2. Example Let $E=\{e_1,e_2,e_3,e_4\}$ be the parameter set and $Z=\{e_1,e_3\}$ and $C=\{e_2,e_3,e_4\}$ be two subsets of E, and $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set. Let $(F,Z)=\{(e_1,\{h_2,h_5\}),(e_3,\{h_1,h_2,h_5\})\}$,$(G,C)=\{(e_2,\{h_1,h_4,h_5\}),(e_3,\{h_2,h_3,h_4\}),(e_4,\{h_3,h_5\})\}$.Then, $(F,Z)\overset{*}{\cap}_{\varepsilon}(G,C)=\{(e_1,\{h_1,h_3,h_4\}),(e_2,\{h_2,h_3\}),(e_3,\{h_2\}),(e_4,\{h_1,h_2,h_4\})\}$.

3.3. Theorem (Algebraic Properties of Operation)

1) The set $S_E(U)$ and $S_Z(U)$, where Z is a fixed subset of E, is closed under $\overset{*}{\cap}_{\varepsilon}$.

2) $[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,C)]\overset{*}{\cap}_{\varepsilon}(H,R)\neq(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,C)\overset{*}{\cap}_{\varepsilon}(H,R)]$.

Proof : Firstly, let's handle the left hand side (LHS). Let $(F,Z)\overset{*}{\cap}_{\varepsilon}(G,C)=(T,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Let $(T,Z\cup C)\overset{*}{\cap}_{\varepsilon}(H,R)=(M,Z\cup C\cup R)$, where for all $\aleph\in Z\cup C\cup R$,

$$M(\aleph)=\begin{cases} T'(\aleph), & \aleph\in(Z\cup C)-R \\ H'(\aleph), & \aleph\in R-(Z\cup C) \\ T(\aleph)\cap H(\aleph), & \aleph\in(Z\cup C)\cap R \end{cases}$$

Thus,

$$M(\aleph)=\begin{cases} F(\aleph), & \aleph\in(Z-C)-R=Z\cap C'\cap R' \\ G(\aleph), & \aleph\in(C-Z)-R=Z'\cap C\cap R' \\ F'(\aleph)\cup G'(\aleph), & \aleph\in(Z\cap C)-R=Z\cap C\cap R' \\ H'(\aleph) & \aleph\in R-(Z\cup C)=Z'\cap C'\cap R \\ F'(\aleph)\cap H(\aleph), & \aleph\in(Z-C)\cap R=Z\cap C'\cap R \\ G'(\aleph)\cap H(\aleph), & \aleph\in(C-Z)\cap R=Z'\cap C\cap R \\ \big(F(\aleph)\cap G(\aleph)\big)\cap H(\aleph), & \aleph\in(Z\cap C)\cap R=Z\cap C\cap R \end{cases}$$

Now let's handle the right hand side (RFS) of the equation, Let $(G,C) \overset{*}{\cap}_{\varepsilon} (H,R)=(K,C\cup R)$. Here, for all $\aleph \in C\cup R$,

$$K(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C-R \\ H'(\aleph), & \aleph\in R-C \\ G(\aleph)\cap H(\aleph), & \aleph\in C\cap R \end{cases}$$

Assume that $(F,Z) \overset{*}{\cap}_{\varepsilon} (K,C\cup R) = (S,Z\cup C\cup R)$, where for all $\aleph \in Z\cup C\cup R$,

$$S(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-(C\cup R) \\ K'(\aleph), & \aleph\in (C\cup R)-Z \\ F(\aleph)\cap K(\aleph), & \aleph\in Z\cap (C\cup R) \end{cases}$$

Thus,

$$S(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-(C\cup R)=Z\cap C'\cap R' \\ G(\aleph), & \aleph\in (C-R)-Z=Z'\cap C\cap R' \\ H(\aleph), & \aleph\in (R-C)-Z=Z'\cap C'\cap R \\ G'(\aleph)\cup H'(\aleph), & \aleph\in (C\cap R)-Z=Z'\cap C\cap R \\ F(\aleph)\cap G'(\aleph), & \aleph\in Z\cap (C-R)=Z\cap C\cap R' \\ F(\aleph)\cap H'(\aleph), & \aleph\in Z\cap (R-C)=Z\cap C'\cap R \\ F(\aleph)\cap (G(\aleph)\cap H(\aleph)), & \aleph\in Z\cap (C\cap R)=Z\cap C\cap R \end{cases}$$

It is seen that $M\neq S$. That is, in the set $S_E(U)$, $\overset{*}{\cap}_{\varepsilon}$ does not have associative property. However, we have the following:

3) $[(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z)] \overset{*}{\cap}_{\varepsilon} (H,Z) = (F,Z) \overset{*}{\cap}_{\varepsilon} [(G,Z) \overset{*}{\cap}_{\varepsilon} (H,Z)]$.

4) $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,C)=(G,C) \overset{*}{\cap}_{\varepsilon} (F,Z)$.

Proof: Firstly, we observe that the parameter set of the soft set on both sides of the equation is $Z\cup C$, and thus the first condition of the soft equality is satisfied. Now let us look at the LHS. Let $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,C)=(H,Z\cup C)$, where for all $\aleph\in Z\cup C$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph\in Z-C \\ G'(\aleph), & \aleph\in C-Z \\ F(\aleph)\cap G(\aleph), & \aleph\in Z\cap C \end{cases}$$

Now let's handle the RHS. Assume that $(G,C) \overset{*}{\cap}_{\varepsilon} (F,Z)=(T,C\cup Z)$, where for all $\aleph\in C\cup Z$,

$$T(\aleph)=\begin{cases} G'(\aleph), & \aleph\in C-Z \\ F'(\aleph), & \aleph\in Z-C \\ G(\aleph)\cap F(\aleph), & \aleph\in C\cap Z \end{cases}$$

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Thus, it is seen that H=T. Similarly, it is easily seen that (F,Z) $\overset{*}{\cap}_{\varepsilon}$(G,Z)=(G,Z) $\overset{*}{\cap}_{\varepsilon}$(F,Z). That is, $\overset{*}{\cap}_{\varepsilon}$ operation is commutative in both $S_E(U)$ and $S_Z(U)$, where Z is a fixed subset of E.

5) (F,Z) $\overset{*}{\cap}_{\varepsilon}$(F,Z)=(F,Z).

Proof: Let (F,Z) $\overset{*}{\cap}_{\varepsilon}$ (F,Z)=(H,Z∪Z), where for all ℵ∈Z, H(ℵ)= F(ℵ)∩F(ℵ)=F(ℵ), and so (H,Z)=(F,Z). That is, $\overset{*}{\cap}_{\varepsilon}$ is idempotent in $S_E(U)$.

6) (F,Z) $\overset{*}{\cap}_{\varepsilon}$ U_Z=U_Z $\overset{*}{\cap}_{\varepsilon}$(F,Z)=(F,Z).

Proof: Let U_Z=(T,Z). Thus, for all ℵ∈Z, T(ℵ)=U. Let (F,Z) $\overset{*}{\cap}_{\varepsilon}$(T, Z) =(H,Z∪Z), where for all ℵ∈Z, H(ℵ)= F(ℵ)∩T(ℵ)= F(ℵ)∩U=F(ℵ), and so (H,Z)=(F,Z). That is, in $S_Z(U)$, the identity element of $\overset{*}{\cap}_{\varepsilon}$ is the soft set.

3.3.1. Theorem*:* By 3.3 Theorem (1), (3), (4), (5) and (6), ($S_Z(U)$,$\tilde{\cap}$) is a commutative, idempotent monoid, that is, a bounded semilattice, whose identity element is U_Z, where Z⊆ E is a fixed set of parameters, Moreover, from 3.3 Theorem (2), $\overset{*}{\cap}_{\varepsilon}$ cannot form a semigroup as it is not associative in S_E (U). Thus, ($S_E(U)$, $\overset{*}{\cap}_{\varepsilon}$) is a groupoid.

7) (F,Z) $\overset{*}{\cap}_{\varepsilon}$ $\emptyset_Z$=$\emptyset_Z$ $\overset{*}{\cap}_{\varepsilon}$(F,Z)= $\emptyset_Z$.

Proof: Let $\emptyset_Z$ =(S,Z). Thus, for all ℵ∈Z, S(ℵ)= ∅. Let (F,Z) $\overset{*}{\cap}_{\varepsilon}$ (S,Z)=(H,Z∪Z), where for all ℵ∈Z, H(ℵ)= F(ℵ)∩S(ℵ)= F(ℵ)∩∅=∅, and so (H,Z)= $\emptyset_Z$. That is, the absorbing element of $\overset{*}{\cap}_{\varepsilon}$ in $S_Z(U)$ is the soft set $\emptyset_Z$.

8) (F,Z) $\overset{*}{\cap}_{\varepsilon}$ $\emptyset_\emptyset$= $\emptyset_\emptyset$ $\overset{*}{\cap}_{\varepsilon}$(F,Z) = $(F,Z)^r$.

Proof: Let $\emptyset_\emptyset$= (K, ∅) and (F,Z) $\overset{*}{\cap}_{\varepsilon}$(K, ∅) = (Q,∪ ∅) = (Q,Z), where for all ℵ∈Z, Q(ℵ)= F'(ℵ), and thus (Q,Z) = $(F,Z)^r$.

9) (F,Z) $\overset{*}{\cap}_{\varepsilon}$$(F,Z)^r$ = $(F,Z)^r$ $\overset{*}{\cap}_{\varepsilon}$(F,Z)= $\emptyset_Z$.

Proof: Let $(F,Z)^r$ =(H,Z), where for all ℵ∈Z, H(ℵ)=F'(ℵ). Let (F,Z) $\overset{*}{\cap}_{\varepsilon}$(H,Z)=(T,Z∪Z), where for all ℵ∈Z, T(ℵ)= F(ℵ)∩H(ℵ)= F(ℵ) ∩F'(ℵ)=∅, and thus (T,Z)= $\emptyset_Z$.

10) $[(F,Z) \overset{*}{\cap}_{\varepsilon} (G,C)]^r = (F,Z)^r \overset{*}{\cup}_{\varepsilon} (G,C)^r$.

Proof: Let $(F,Z) \overset{*}{\cap} (G,C) = (H, Z\cup C)$, where for all $\aleph \in Z\cup C$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph \in Z-C \\ G'(\aleph), & \aleph \in C-Z \\ F(\aleph)\cap G(\aleph), & \aleph \in Z\cap C \end{cases}$$

Let $(H, Z\cup C)^r = (T, Z\cup C)$, where for all $\aleph \in Z\cup C$,

$$T(\aleph)=\begin{cases} F(\aleph), & \aleph \in Z-C \\ G(\aleph), & \aleph \in C-Z \\ F'(\aleph) \cup G'(\aleph), & \aleph \in Z\cap C \end{cases}$$

Hence, $(T, Z\cup C) = (F,Z)^r \overset{*}{\cup}_{\varepsilon} (G,C)^r$.

11) $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z) = U_Z \Leftrightarrow (F,Z) = (G,Z) = U_Z$.

Proof: Let $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z) = (T, Z\cup Z)$, where for all $\aleph \in Z$, $T(\aleph) = F(\aleph) \cap G(\aleph)$. Since $(T,Z) = U_Z$, $T(\aleph) = U$ for all $\aleph \in Z$, and thus, $F(\aleph) \cap G(\aleph) = U$ for all $\aleph \in Z$. So, $F(\aleph) = G(\aleph) = U$ for all $\aleph \in Z$. Hence, $(F,Z) = (G,Z) = U_Z$.

12) $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z) = \emptyset_Z \Leftrightarrow (G,Z) \tilde{\subseteq} (F,Z)^r$ and $(F,Z) \tilde{\subseteq} (G,Z)^r$.

Proof: Let $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z) = (T,Z)$, where for all $\aleph \in Z$, $T(\aleph) = F(\aleph) \cap G(\aleph)$. Since $(T,Z) = \emptyset_Z$, thus, for all $\aleph \in Z$, $T(\aleph) = \emptyset$. Hence, for all $\aleph \in Z$, $T(\aleph) = F(\aleph) \cap G(\aleph) = \emptyset \Leftrightarrow G(\aleph) \subseteq F'(\aleph)$. Therefore, $(G,Z) \tilde{\subseteq} (F,Z)^r$.

Similarly, if $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z) = \emptyset_Z$, then $(F,Z) \tilde{\subseteq} (G,Z)^r$ can be shown similarly. Conversely, let $(G,Z) \tilde{\subseteq} (F,)^r$. Thus, for all $\aleph \in Z$, $F(\aleph) \cap G(\aleph) = \emptyset$. So, $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z) = \emptyset_Z$.

13) $\emptyset_Z \tilde{\subseteq} (F,Z) \overset{*}{\cap}_{\varepsilon} (G,C)$, $\emptyset_C \tilde{\subseteq} (F,Z) \overset{*}{\cap}_{\varepsilon} (G,C)$, $\emptyset_{Z\cup C} \tilde{\subseteq} (F,Z) \overset{*}{\cap}_{\varepsilon} (G,C)$, $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,C) \tilde{\subseteq} U_{Z\cup C}$.

14) $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z) \tilde{\subseteq} (F,Z)$ and $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z) \tilde{\subseteq} (G,Z)$.

Proof: Let $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z) = (H, Z \cup Z)$, where for all $\aleph \in Z$, Since for all $\aleph \in Z$, $H(\aleph) = F(\aleph) \cap G(\aleph) \subseteq F(\aleph)$, $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z) \tilde{\subseteq} (F,Z)$. Similarly, for all $\aleph \in Z$, $H(\aleph) = F(\aleph) \cap G(\aleph) \subseteq G(\aleph)$. Thus, $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z) \tilde{\subseteq} (G,Z)$.

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15) $(F,Z) \widetilde{\subseteq} (G,Z) \Leftrightarrow (F,Z) \overset{*}{\cap_\varepsilon} (G,Z) = (F,Z)$.

Proof: Let $(F,Z) \widetilde{\subseteq} (G,Z)$. Then for all $\aleph \in Z$, $F(\aleph) \subseteq G(\aleph)$. Let $(F,Z) \overset{*}{\cap_\varepsilon} (G,Z)=(H,Z)$, where for all $\aleph \in Z$, $H(\aleph)=F(\aleph) \cap G(\aleph)$. Thus, for all $\aleph \in Z$, $H(\aleph)=F(\aleph) \cap G(\aleph)=F(\aleph)$ implying that $(F,Z) \overset{*}{\cap_\varepsilon} (G,Z)=(F,Z)$. Conversely, let $(F,Z) \overset{*}{\cap_\varepsilon} (G,Z)=(F,Z)$. Hence, $F(\aleph) \cap G(\aleph)=F(\aleph)$ and thus, $F(\aleph) \subseteq G(\aleph)$, for all $\aleph \in Z$. Thereby, $(F,Z) \widetilde{\subseteq} (G,Z)$.

16) $(F,Z) \overset{*}{\cap_\varepsilon} (G,C) \widetilde{\subseteq} (F,Z) \overset{*}{\cup_\varepsilon} (G,C)$.

Proof: Let $(F,Z) \overset{*}{\cap_\varepsilon} (G,C)=(H,Z \cup C)$, where $\aleph \in Z \cup C$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph \in Z-C \\ G'(\aleph), & \aleph \in C-Z \\ F(\aleph) \cap G(\aleph), & \aleph \in Z \cap C \end{cases}$$

Let $(F,Z) \overset{*}{\cup_\varepsilon} (G,C)=(T,Z \cup C)$, where $\aleph \in Z \cup C$,

$$T(\aleph)=\begin{cases} F'(\aleph), & \aleph \in Z-C \\ G'(\aleph), & \aleph \in C-Z \\ F(\aleph) \cup G(\aleph), & \aleph \in Z \cap C \end{cases}$$

Here, for all $\aleph \in Z-C$, $H(\aleph)=F'(\aleph) \subseteq F'(\aleph)=T(\aleph)$, for all $\aleph \in C-Z$, $H(\aleph)=G'(\aleph) \subseteq G'(\aleph)=T(\aleph)$, and for all $\aleph \in Z \cap C$, $H(\aleph) = F(\aleph) \cap G(\aleph) \subseteq F(\aleph) \cup G(\aleph)=T(\aleph)$. Thus, $(F,Z) \overset{*}{\cap_\varepsilon} (G,C) \widetilde{\subseteq} (F,Z) \overset{*}{\cup_\varepsilon} (G,C)$.

17) $(F,Z) \overset{*}{\cap_\varepsilon} (G,C) = (F,C) \overset{*}{\cup_\varepsilon} (G,C) \Leftrightarrow (F,Z \cap C) = (G,Z \cap C)$.

Proof: Let $(F,Z) \overset{*}{\cap_\varepsilon} (G,C) = (F,Z) \overset{*}{\cup_\varepsilon} (G,C)$ and $(F,Z) \overset{*}{\cap_\varepsilon} (G,C) = (H,Z \cup C)$, where for all $\aleph \in Z \cup C$,

$$H(\aleph)=\begin{cases} F'(\aleph), & \aleph \in Z-C \\ G'(\aleph), & \aleph \in C-Z \\ F(\aleph) \cap G(\aleph), & \aleph \in Z \cap C \end{cases}$$

Let $(F,Z) \overset{*}{\cup_\varepsilon} (G,C) = (K,Z \cup C)$, where for all $\aleph \in Z \cup C$,

$$K(\aleph)=\begin{cases} F'(\aleph), & \aleph \in Z-C \\ G'(\aleph), & \aleph \in C-Z \\ F(\aleph) \cup G(\aleph), & \aleph \in C \cap Z \end{cases}$$

Since $(H,Z \cup C) = (K,Z \cup C)$, $F'(\aleph)=F'(\aleph)$ for all $\aleph \in Z-C$, $G'(\aleph)=G'(\aleph)$ for all $\aleph \in C-Z$, and $F(\aleph) \cap G(\aleph)= F(\aleph) \cup G(\aleph)$, for all $\aleph \in Z \cap C$. Thus, $F(\aleph) = G(\aleph)$ for all $\aleph \in Z \cap C$, implying that $(F,Z \cap C) = (G, Z \cap C)$.

Conversely, let (F,Z∩C)=(G,Z∩C). Hence, for all $\aleph \in Z \cap C$, $F(\aleph)=G(\aleph)$. Moreover, for all $\aleph\in Z-C$, $H(\aleph) = F'(\aleph)=F'(\aleph)=K(\aleph)$; for all $\aleph\in C-Z$, $H(\aleph)=G'(\aleph)=G'(\aleph) = K(\aleph)$, and and so for all $\aleph \in Z$, $H(\aleph)=K(\aleph)$ implying that $(F,Z) \overset{*}{\cap}_{\varepsilon} (G,C) = (F,Z) \overset{*}{\cup}_{\varepsilon} (G,C)$.

18) If $(F,Z) \tilde{\subseteq} (G,Z)$, then $(F,Z) \overset{*}{\cap}_{\varepsilon} (H,Z) \tilde{\subseteq} (G,Z) \overset{*}{\cap}_{\varepsilon} (H,Z)$.

Proof: Let $(F,Z)\tilde{\subseteq} (G,Z)$. Hence, for all $\aleph\in Z$, $F(\aleph)\subseteq G(\aleph)$. Let $(F,Z) \overset{*}{\cap}_{\varepsilon} (H,Z)=(W,Z)$, where for all $\aleph\in Z$, $W(\aleph)=F(\aleph)\cap H(\aleph)$. Let $(G,Z) \overset{*}{\cap}_{\varepsilon} (H,Z) = (L,Z)$. Thus, for all $\aleph\in Z$, $L(\aleph)=G(\aleph)\cap H(\aleph)$. Thus, for all $\aleph\in Z$, $W(\aleph)=F(\aleph)\cap H(\aleph) \subseteq G(\aleph)\cap H(\aleph)=L(\aleph)$. Hence, $(F,Z) \overset{*}{\cap}_{\varepsilon} (H,Z) \tilde{\subseteq} (G,Z) \overset{*}{\cap}_{\varepsilon} (H,Z)$.

19) If $(F,Z) \overset{*}{\cap}_{\varepsilon} (H,Z) \tilde{\subseteq} (G,Z) \overset{*}{\cap}_{\varepsilon} (H,Z)$, then $(F,Z)\tilde{\subseteq}(G,Z)$ need not have to be true. That is, the converse of 3.3. Theorem (18) is not true.

Proof: Let us give an example to show that the converse of 3.3. Theorem (18) is not true. Let $E=\{e_1,e_2,e_3,e_4,e_5\}$ be the parameter set, $A=C=\{e_1,e_3\}$ be the subset of E, and $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set.

Let $(F,Z)=\{(e_1,\{h_2,h_5\}),(e_3,\{h_1,h_2,h_5\})\}$,$(G,Z)=\{(e_1,\{h_2\}),(e_3,\{h_1,h_2\})\}$,$(H,Z)=\{(e_1,\emptyset),(e_3,\emptyset)\}$ be soft sets over U. Let $(F,Z) \overset{*}{\cap}_{\varepsilon} (H,Z)=(L,Z)$, then $(L,Z)=\{(e_1,\emptyset),(e_3,\emptyset)\}$ and let $(G,Z) \overset{*}{\cap}_{\varepsilon} (H,Z)=(K,Z)$, thus $(K,Z)=\{(e_1,\emptyset),(e_3,\emptyset)\}$. Hence, $(F,Z) \overset{*}{\cap}_{\varepsilon} (H,Z) \tilde{\subseteq} (G,Z) \overset{*}{\cap}_{\varepsilon} (H,Z)$ but (F,Z) is not a soft subset of (G,Z).

20) If $(F,Z) \tilde{\subseteq} (G,C)$ and $(K,Z) \tilde{\subseteq} (L,C)$, then $(F,Z) \overset{*}{\cap}_{\varepsilon} (K,Z) \tilde{\subseteq} (G,C) \overset{*}{\cap}_{\varepsilon} (L,C)$.

Proof: Let $(F,Z)\tilde{\subseteq} (G,C)$ and $(K,Z) \tilde{\subseteq} (L,C)$. Hence, $Z\subseteq C$ and for all $\aleph\in Z$, $F(\aleph)\subseteq G(\aleph)$ and $K(\aleph)\subseteq L(\aleph)$. Let $(F,Z) \overset{*}{\cap}_{\varepsilon} (K,Z) = (W,Z)$. Thus, for all $\aleph\in Z$, $W(\aleph)=F(\aleph)\cap K(\aleph)$. Let $(G,C) \overset{*}{\cap}_{\varepsilon} (L,C) = (S,C)$. Thus, for all $\aleph\in C$, $S(\aleph)=G(\aleph)\cap L(\aleph)$. Since $W(\aleph)=F(\aleph)\cap K(\aleph) \subseteq G(\aleph)\cap L(\aleph)=S(\aleph)$ for all $\aleph\in Z$, $(F,Z) \overset{*}{\cap}_{\varepsilon} (K,Z) \tilde{\subseteq} (G,C) \overset{*}{\cap}_{\varepsilon} (L,C)$.

21) $(F,Z) \overset{*}{\cap}_{\varepsilon} ([(F,Z) \overset{*}{\cup}_{\varepsilon} (G,Z)]= (F,Z)$ and $(F,Z) \overset{*}{\cup}_{\varepsilon} \left[(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z)\right] = (F,Z)$ (The absorbtion laws)

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Proof: Let $(F,Z) \overset{*}{\cup}_{\varepsilon} (G,Z)=(T,Z)$, where for all $\aleph\in Z$, $T(\aleph)= F(\aleph)\cap G(\aleph)$. Let $(F,Z) \overset{*}{\cap}_{\varepsilon} (T,Z)=(M,Z)$, where for all $\aleph\in Z$, $M(\aleph) = F(\aleph)\cap T(\aleph)$. Thus, $M(\aleph)=F(\aleph) \cup [F(\aleph) \cap G(\aleph)]$, for all $\aleph\in Z$. Hence, $M(\aleph)=F(\aleph)$, for all $\aleph\in Z$. Thus, $(M,Z)=(F,Z)$. Similarly, $(F,Z) \overset{*}{\cup}_{\varepsilon} \left[(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z)\right] = (F,Z)$ can be shown. Here, if $\overset{*}{\cup}_{\varepsilon}$ is replaced by the restricted union operation or the extended union operation, it is evident that the 3.3. Theorem 3.3. (24) holds again in $S_Z(U)$ with Z being a fixed subset of the universal set E, since these operations coincide with each other in the collection $S_Z(U)$.

3.3.2. Theorem $(S_Z(U), {}^r, \overset{*}{\cap}_{\varepsilon}, U_Z)$ is an MV-algebra.

Proof: Let's show that $(S_Z(U), {}^r, \overset{*}{\cap}_{\varepsilon}, U_Z)$ satisfies the MV-algebra conditions.

(MV1) $(S_Z(U), \overset{*}{\cap}_{\varepsilon}, U_Z)$ is a commutative monoid.

(MV2) $((F,Z)^r)^r =(F,Z)$ (Ali et al. 2011).

(MV3) $(U_Z)^r \overset{*}{\cap}_{\varepsilon}(F,Z)= \emptyset_Z \overset{*}{\cap}_{\varepsilon}(F,Z) = \emptyset_Z = (U_Z)^r$.

(MV4) $[(F,Z)^r \overset{*}{\cap}_{\varepsilon}(G,Z)]^r \overset{*}{\cap}_{\varepsilon}(G,Z)=((G,Z)^r \overset{*}{\cap}_{\varepsilon}(F,Z)^r)^r \overset{*}{\cap}_{\varepsilon}(F,Z)$.

3.4. Theorem The complementary extended intersection operation has the following distributions over other soft set operations:

3.4.1. Theorem The complementary extended intersection operation has the following distributions over restricted soft set operations: (The followings hold where $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$)

i) LHS Distributions of the Complementary Extended Intersection Operation over Restricted Soft Set Operations:

1) $(F,Z)\overset{*}{\cap}_{\varepsilon} [(G,C)\cap_R(H,R)]=[(F,Z)\overset{*}{\cap}_{\varepsilon} (G,C)]\cup_R[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]$,

2) $(F,Z)\overset{*}{\cap}_{\varepsilon} [(G,C)\cup_R(H,R)]=[(F,Z)\overset{*}{\cap}_{\varepsilon} (G,C)]\cup_R[(F,Z)\overset{*}{\cap}_{\varepsilon} (H,R)]$.

3) $(F,Z) \overset{*}{\cap}_{\varepsilon} [(G,C)*_R(H,R)]=[(F,Z)\overset{*}{\backslash}_{\varepsilon} (G,C)]\cup_R[(F,Z)\overset{*}{\backslash}_{\varepsilon} (H,R)]$.

4) $(F,Z) \overset{*}{\cap}_{\varepsilon} [(G,C)\theta_R(H,R)]=[(F,Z)\overset{*}{\backslash}_{\varepsilon} (G,C)]\cap_R[(F,Z)\overset{*}{\backslash}_{\varepsilon} (H,R)]$.

ii) RHS Distribution of Complementary Extended Intersection Operation over Restricted Soft Set Operations

1) $[(F,Z)\cup_R(G,C)]\overset{*}{\cap}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]\cup_R[(G,C)\overset{*}{\cap}_{\varepsilon}(H,R)]$.

2) $[(F,Z)\cap_R(G,C)]\overset{*}{\cap}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]\cap_R[(G,C)\overset{*}{\cap}_{\varepsilon}(H,R)]$.

3) $[(F,Z)*_R(G,C)]\overset{*}{\cap}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma}_{\varepsilon}(H,R)]\cup_R[(G,C)\overset{*}{\gamma}_{\varepsilon}(H,R)]$.

4) $[(F,Z)\theta_R(G,C)]\overset{*}{\cap}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma}_{\varepsilon}(H,R)]\cap_R[(G,C)\overset{*}{\gamma}_{\varepsilon}(H,R)]$.

3.4.1.1. Corollary: Considering the distributions in 3.4.1 Theorem, we have:

- $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,Z)\cap_R(H,Z)]=[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,Z)]\cap_R[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.
- $[(F,Z)\cap_R(G,Z)]\overset{*}{\cap}_{\varepsilon}(H,Z)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]\cap_R[(G,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.
- $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,Z)\cup_R(H,Z)]=[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,Z)]\cup_R[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.
- $[(F,Z)\cup_R(G,Z)]\overset{*}{\cap}_{\varepsilon}(H,Z)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]\cup_R[(G,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.
- $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,Z)\Delta_R(H,Z)]=[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,Z)]\Delta_R[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.
- $[(F,Z)\Delta_R(G,Z)]\overset{*}{\cap}_{\varepsilon}(H,Z)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]\Delta_R[(G,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.

3.4.1.2. Theorem: $(S_Z(U),\cap_R,\overset{*}{\cap}_{\varepsilon})$ is a commutative, idempotent semiring without zero but with unity.

3.4.1.3 Theorem: $(S_Z(U),\cup_R,\overset{*}{\cap}_{\varepsilon})$ is a commutative, idempotent hemiring with unity.

3.4.1.4. Theorem: $(S_Z(U),\Delta_R,\overset{*}{\cap}_{\varepsilon})$ is a Boolean Ring, and also $(S_Z(U),\Delta_R,\overset{*}{\cap}_{\varepsilon})$ is a commutative, multiplicative idempotent hemiring with unity.

3.4.1.5. Theorem: $(S_Z(U),\emptyset_Z,U_Z,\cup_R,\overset{*}{\cap}_{\varepsilon})$ Boolean, De Morgan, Kleene and Stone algebra.

3.4.2. Theorem The following distributions of the complementary extended intersection operation over extended soft set operations hold:

i)LHS Distributions of the Complementary Extended Intersection Operation on Extended Soft Set Operations:

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The followings hold where $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$.

1) $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,C)*_{\varepsilon}(H,R)]=[(F,Z)\overset{*}{\backslash}_{\varepsilon}(G,C)]\cup_{\varepsilon}[(F,Z)\overset{*}{\backslash}_{\varepsilon}(H,R)]$.

2) $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,C)\cap_{\varepsilon}(H,R)]=[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,C)]\cap_{\varepsilon}[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]$.

3) $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,C)\cup_{\varepsilon}(H,R)]=[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,C)]\cup_{\varepsilon}[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]$.

4) $F,Z)\overset{*}{\cap}_{\varepsilon}[(G,C)\theta_{\varepsilon}(H,R)]=[(F,Z)\overset{*}{\backslash}_{\varepsilon}(G,C)]\cap_{\varepsilon}[(F,Z)\overset{*}{\backslash}_{\varepsilon}(H,R)]$.

ii) RHS Distributions of Complementary Extended Intersection Operation over Extended Soft Set Operations

1) $[(F,Z)\cup_{\varepsilon}(G,C)]\overset{*}{\cap}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]\cup_{\varepsilon}[(G,C)\overset{*}{\cap}_{\varepsilon}(H,R)]$.

2) $[(F,Z)\cap_{\varepsilon}(G,C)]\overset{*}{\cap}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]\cap_{\varepsilon}[(G,C)\overset{*}{\cap}_{\varepsilon}(H,R)]$.

3) $[(F,Z)\theta_{\varepsilon}(G,C)]\overset{*}{\cap}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma}_{\varepsilon}(H,R)]\cap_{\varepsilon}[(G,C)\overset{*}{\gamma}_{\varepsilon}(H,R)]$.

4) $[(F,Z)*_{\varepsilon}(G,C)]\overset{*}{\cap}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma}_{\varepsilon}(H,R)]\cup_{\varepsilon}[(G,C)\overset{*}{\gamma}_{\varepsilon}(H,R)]$.

3.4.2.1. Corollary: Considering the distributions in 3.4.2. Theorem, we have:

- $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,Z)\cap_{\varepsilon}(H,Z)]=[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,Z)]\cap_{\varepsilon}[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.
- $[(F,Z)\cap_{\varepsilon}(G,Z)]\overset{*}{\cap}_{\varepsilon}(H,Z)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]\cap_{\varepsilon}[(G,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.
- $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,Z)\cup_{\varepsilon}(H,Z)]=[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,Z)]\cup_{\varepsilon}[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.
- $[(F,Z)\cup_{\varepsilon}(G,Z)]\overset{*}{\cap}_{\varepsilon}(H,Z)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]\cup_{\varepsilon}[(G,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.
- $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,Z)\Delta_{\varepsilon}(H,Z)]=[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,Z)]\Delta_{\varepsilon}[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.
- $[(F,Z)\Delta_{\varepsilon}(G,Z)]\overset{*}{\cap}_{\varepsilon}(H,Z)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]\Delta_{\varepsilon}[(G,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.

3.4.2.2. Theorem: $(S_Z(U),\cap_{\varepsilon},\overset{*}{\cap}_{\varepsilon})$ is a commutative, idempotent semiring without zero but with unity.

3.4.2.3 Theorem: $(S_Z(U),\cup_{\varepsilon},\overset{*}{\cap}_{\varepsilon})$ is a commutative, idempotent hemiring with unity.

3.4.2.4. Theorem: $(S_Z(U),\Delta_\varepsilon,\overset{*}{\cap}_{\varepsilon})$ is a Boolean Ring, annd also a commutative, multiplicative idempotent hemiring with unity.

3.4.2.5. Theorem: $(S_Z(U),\emptyset_Z,U_Z,\cup_\varepsilon,\overset{*}{\cap}_{\varepsilon})$ Boolean, De Morgan, Kleene and Stone algebra.

3.4.3. Theorem: The following distributions of the complementary extended intersection operation over complementary extended operations hold:

i) LHS Distributions of Complementary Extended Intersection Operations over Complementary Extended Soft Set Operations

The followings hold where $(Z\Delta C)\cap R=Z\cap C\cap R'=\emptyset$.

1) $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,C)\overset{*}{\cap}_{\varepsilon}(H,R)]=[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,C)]\overset{*}{\cap}_{\varepsilon}[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]$.

2) $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,C)\overset{*}{\cup}_{\varepsilon}(H,R)]=[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,C)]\overset{*}{\cup}_{\varepsilon}[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]$.

3) $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,C)\overset{*}{*}_{\varepsilon}(H,R)]=[(F,Z)\overset{*}{\backslash}_{\varepsilon}(G,C)]\overset{*}{\cap}_{\varepsilon}[(F,Z)\overset{*}{\backslash}_{\varepsilon}(H,R)]$.

4) $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,C)\overset{*}{\theta}_{\varepsilon}(H,R)]=[(F,Z)\overset{*}{\backslash}_{\varepsilon}(G,C)]\overset{*}{\cap}_{\varepsilon}[(F,Z)\overset{*}{\backslash}_{\varepsilon}(H,R)]$.

ii) RHS Distributions of Complementary Extended Intersection Operation over Complementary Extended Operations

1) $F,Z)\overset{*}{\theta}_{\varepsilon}(G,C)]\overset{*}{\cap}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\backslash}_{\varepsilon}(H,R)]\overset{*}{\cap}_{\varepsilon}[(G,C)\overset{*}{\backslash}_{\varepsilon}(H,R)]$,

2) $[(F,Z)\overset{*}{\cup}_{\varepsilon}(G,C)]\overset{*}{\cap}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]\overset{*}{\cup}_{\varepsilon}[(G,C)\overset{*}{\cap}_{\varepsilon}(H,R)]$.

3) $[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,C)]\overset{*}{\cap}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,R)]\overset{*}{\cap}_{\varepsilon}[(G,C)\overset{*}{\cap}_{\varepsilon}(H,R)]$.

4) $[(F,Z)\overset{*}{*}_{\varepsilon}(G,C)]\overset{*}{\cap}_{\varepsilon}(H,R)=[(F,Z)\overset{*}{\gamma}_{\varepsilon}(H,R)]\overset{*}{\cup}_{\varepsilon}[(G,C)\overset{*}{\gamma}_{\varepsilon}(H,R)]$.

3.4.3.1. Corollary: Considering the distributions in 3.4.3 Theorem, we have:

- $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]=[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,Z)]\overset{*}{\cap}_{\varepsilon}[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.
- $[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,Z)]\overset{*}{\cap}_{\varepsilon}H,Z)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]\overset{*}{\cap}_{\varepsilon}[(G,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.
- $(F,Z)\overset{*}{\cap}_{\varepsilon}[(G,Z)\overset{*}{\cup}_{\varepsilon}(H,Z)]=[(F,Z)\overset{*}{\cap}_{\varepsilon}(G,Z)]\overset{*}{\cup}_{\varepsilon}[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.
- $[(F,Z)\overset{*}{\cup}_{\varepsilon}(G,Z)]\overset{*}{\cap}_{\varepsilon}(H,Z)=[(F,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]\overset{*}{\cup}_{\varepsilon}[(G,Z)\overset{*}{\cap}_{\varepsilon}(H,Z)]$.

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3.4.3.2. Theorem: $(S_Z(U), \overset{*}{\cap}_\varepsilon, \overset{*}{\cap}_\varepsilon)$ is a commutative, idempotent semiring without zero but with unity.

3.4.3.3 Theorem: $(S_Z(U), \overset{*}{\cup}_\varepsilon, \overset{*}{\cap}_\varepsilon)$ is a commutative, idempotent hemiring with unity.

3.4.3.4. Theorem: $(S_Z(U), \overset{*}{\Delta}_\varepsilon, \overset{*}{\cap}_\varepsilon)$ is a Boolean Ring, and also a commutative, multiplicative idempotent hemiring with unity.

3.4.2.5. Theorem: $(S_Z(U), \emptyset_Z, U_Z, \overset{*}{\cup}_\varepsilon, \overset{*}{\cap}_\varepsilon)$ Boolean, De Morgan, Kleene and Stone algebra.

3.4.4. Theorem The following distributions of the complementary extended intersection operation over soft binary piecewise operations hold:

i) LHS Distributions of the Complementary Extended Intersection Operation over Soft Binary Pievewise Operations:

1) $(F,Z) \overset{*}{\cap}_\varepsilon [(G,C) \overset{\sim}{\cap} (H,R)] = [(F,Z) \overset{*}{\cap}_\varepsilon (G,C)] \overset{\sim}{\cap} [(F,Z) \overset{*}{\cap}_\varepsilon (H,R)]$.

2) $(F,Z) \overset{*}{\cap}_\varepsilon [(G,C) \overset{\sim}{\cup} (H,R)] = [(F,Z) \overset{*}{\cap}_\varepsilon (G,C)] \overset{\sim}{\cup} [(F,Z) \overset{*}{\cap}_\varepsilon (H,R)]$.

3) $(F,Z) \overset{*}{\cap}_\varepsilon [(G,C) \overset{\sim}{*} (H,R)] = [(F,Z) \overset{*}{\backslash}_\varepsilon (G,C)] \overset{\sim}{\cup} [(F,Z) \overset{*}{\backslash}_\varepsilon (H,R)]$.

4) $(F,Z) \overset{*}{\cap}_\varepsilon [(G,C) \overset{\sim}{\theta} (H,R)] = [(F,Z) \overset{*}{\backslash}_\varepsilon (G,C)] \overset{\sim}{\cap} [(F,Z) \overset{*}{\backslash}_\varepsilon (H,R)]$.

ii) RHS Distributions of the Complementary Extended Intersection Operation over Soft Binary Piecewise Operations

1) $[(F,Z) \overset{\sim}{\theta} (G,C)] \overset{*}{\cap}_\varepsilon (H,R) = [(F,Z) \overset{*}{\gamma}_\varepsilon (H,R)] \overset{\sim}{\cap} [(G,C) \overset{*}{\gamma}_\varepsilon (H,R)]$.

2) $[(F,Z) \overset{\sim}{\cup} (G,C)] \overset{*}{\cap}_\varepsilon (H,R) = [(F,Z) \overset{*}{\cap}_\varepsilon (H,R)] \overset{\sim}{\cup} [(G,C) \overset{*}{\cap}_\varepsilon (H,R)]$.

3) $(F,Z) \overset{\sim}{\cap} (G,C)] \overset{*}{\cap} (H,R) = [(F,Z) \overset{*}{\cap}_\varepsilon (H,R)] \overset{\sim}{\cap} [(G,C) \overset{*}{\cap}_\varepsilon (H,R)]$

4) $[(F,Z) \overset{\sim}{*} (G,C)] \overset{*}{\cap}_\varepsilon (H,R) = [(F,Z) \overset{*}{\gamma}_\varepsilon (H,R)] \overset{\sim}{\cup} [(G,C) \overset{*}{\gamma}_\varepsilon (H,R)]$.

3.4.4.4.1. Corollary: Considering the distributions in 3.4.4. Theorem, we have:

- $(F,Z)\overset{*}{\cap_\varepsilon}[(G,Z)\,\tilde{\cap}\,(H,Z)]=[(F,Z)\overset{*}{\cap_\varepsilon}(G,Z)]\,\tilde{\cap}\,[(F,Z)\overset{*}{\cap_\varepsilon}(H,Z)]$.
- $[(F,Z)\,\tilde{\cap}\,(G,Z)]\overset{*}{\cap_\varepsilon}H,Z)=[(F,Z)\overset{*}{\cap_\varepsilon}(H,Z)]\,\tilde{\cap}\,[(G,Z)\overset{*}{\cap_\varepsilon}(H,Z)]$.
- $(F,Z)\overset{*}{\cap_\varepsilon}[(G,Z)\,\tilde{\cup}\,(H,Z)]=[(F,Z)\overset{*}{\cap_\varepsilon}(G,Z)]\,\tilde{\cup}\,[(F,Z)\overset{*}{\cap_\varepsilon}(H,Z)]$.
- $[(F,Z)\,\tilde{\cup}\,(G,Z)]\overset{*}{\cap_\varepsilon}(H,Z)=[(F,Z)\overset{*}{\cap_\varepsilon}(H,Z)]\,\tilde{\cup}\,[(G,Z)\overset{*}{\cap_\varepsilon}(H,Z)]$.
- $(F,Z)\overset{*}{\cap_\varepsilon}[(G,Z)\,\tilde{\Delta}\,(H,Z)]=[(F,Z)\overset{*}{\cap_\varepsilon}(G,Z)]\,\tilde{\Delta}\,[(F,Z)\overset{*}{\cap_\varepsilon}(H,Z)]$.
- $[(F,Z)\,\tilde{\Delta}\,(G,Z)]\overset{*}{\cap_\varepsilon}(H,Z)=[(F,Z)\overset{*}{\cap_\varepsilon}(H,Z)]\,\tilde{\Delta}\,[(G,Z)\overset{*}{\cap_\varepsilon}(H,Z)]$.

3.4.4.2. Theorem: $(S_Z(U),\tilde{\cap},\overset{*}{\cap_\varepsilon})$ is a commutative, idempotent semiring without zero but with unity.

3.4.4.3 Theorem: $(S_Z(U),\tilde{\cup},\overset{*}{\cap_\varepsilon})$ is a commutative, idempotent hemiring with unity.

3.4.4.4. Theorem: $(S_Z(U),\tilde{\Delta},\overset{*}{\cap_\varepsilon})$ is a Boolean Ring, and also a commutative, multiplicative idempotent hemiring with unity.

3.4.4.5. Theorem: $(S_Z(U),\emptyset_Z,U_Z,\tilde{\cup},\overset{*}{\cap_\varepsilon})$ Boolean, De Morgan, Kleene and Stone algebra.

3.4.5. Theorem: The following distributions of the complementary extended intersection operation over the complementary soft binary piecewise operations exist:

i) LHS Distribution of the Complementary Extended Intersection Operation over Complementary Soft Binary Piecewise Operations

1) If $(F,Z)\overset{*}{\cap_\varepsilon}[(G,C)\underset{\cap}{\overset{*}{\sim}}(H,R)]=[(F,Z)\overset{*}{\cap_\varepsilon}(G,C)]\underset{\cap}{\overset{*}{\sim}}[(F,Z)\overset{*}{\cap_\varepsilon}(H,R)]$.

2) $(F,Z)\overset{*}{\cap_\varepsilon}[(G,C)\underset{\cup}{\overset{*}{\sim}}(H,R)]=[(F,Z)\overset{*}{\cap_\varepsilon}(G,C)]\underset{\cup}{\overset{*}{\sim}}[(F,Z)\overset{*}{\cap_\varepsilon}(H,R)]$.

3) $(F,Z)\overset{*}{\cap_\varepsilon}[(G,C)\underset{*}{\overset{*}{\sim}}(H,R)]=[(F,Z)\overset{*}{\backslash_\varepsilon}(G,C)]\underset{\cup}{\overset{*}{\sim}}[(F,Z)\overset{*}{\backslash_\varepsilon}(H,R)]$.

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4) $(F,Z) \overset{*}{\cap}_{\varepsilon} [(G,C) \underset{\theta}{\overset{*}{\sim}} (H,R)] = [(F,Z) \overset{*}{\setminus}_{\varepsilon} (G,C)] \underset{\cap}{\overset{*}{\sim}} [(F,Z) \overset{*}{\setminus}_{\varepsilon} (H,R)]$.

ii) RHS Distributions of Complementary Extended Intersection Operation over Complementary Soft Binary Piecewise Operations

1) $[(F,Z) \underset{\theta}{\overset{*}{\sim}} (G,C)] \overset{*}{\cap}_{\varepsilon} (H,R) = [(F,Z) \overset{*}{\gamma}_{\varepsilon} (H,R)] \underset{\cap}{\overset{*}{\sim}} [(G,C) \overset{*}{\gamma}_{\varepsilon} (H,R)]$.

2) $[(F,Z) \underset{\cup}{\overset{*}{\sim}} (G,C)] \overset{*}{\cap}_{\varepsilon} (H,R) = [(F,Z) \overset{*}{\cap}_{\varepsilon} (H,R)] \underset{\cup}{\overset{*}{\sim}} [(G,C) \overset{*}{\cap}_{\varepsilon} (H,R)]$.

3) $[(F,Z) \underset{\cap}{\overset{*}{\sim}} (G,C)] \overset{*}{\cap}_{\varepsilon} (H,R) = [(F,Z) \overset{*}{\cap}_{\varepsilon} (H,R)] \underset{\cap}{\overset{*}{\sim}} [(G,C) \overset{*}{\cap}_{\varepsilon} (H,R)]$.

4) $[(F,Z) \underset{*}{\overset{*}{\sim}} (G,C)] \overset{*}{\cap}_{\varepsilon} (H,R) = [(F,Z) \overset{*}{\gamma}_{\varepsilon} (H,R)] \underset{\cup}{\overset{*}{\sim}} [(G,C) \overset{*}{\gamma}_{\varepsilon} (H,R)]$.

3.4.5.1. Corollary: Considering the distributions in 3.4.5. Theorem, we have:

- $(F,Z) \overset{*}{\cap}_{\varepsilon} [(G,Z) \underset{\cap}{\overset{*}{\sim}} (H,Z)] = [(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z)] \underset{\cap}{\overset{*}{\sim}} [(F,Z) \overset{*}{\cap}_{\varepsilon} (H,Z)]$.
- $[(F,Z) \underset{\cap}{\overset{*}{\sim}} (G,Z)] \overset{*}{\cap}_{\varepsilon} H,Z) = [(F,Z) \overset{*}{\cap}_{\varepsilon} (H,Z)] \underset{\cap}{\overset{*}{\sim}} [(G,Z) \overset{*}{\cap}_{\varepsilon} (H,Z)]$.
- $(F,Z) \overset{*}{\cap}_{\varepsilon} [(G,Z) \underset{\cup}{\overset{*}{\sim}} (H,Z)] = [(F,Z) \overset{*}{\cap}_{\varepsilon} (G,Z)] \underset{\cup}{\overset{*}{\sim}} [(F,Z) \overset{*}{\cap}_{\varepsilon} (H,Z)]$.
- $[(F,Z) \underset{\cup}{\overset{*}{\sim}} (G,Z)] \overset{*}{\cap}_{\varepsilon} (H,Z) = [(F,Z) \overset{*}{\cap}_{\varepsilon} (H,Z)] \underset{\cup}{\overset{*}{\sim}} [(G,Z) \overset{*}{\cap}_{\varepsilon} (H,Z)]$

3.4.5.2. Corollary: $(S_Z(U), \underset{\cap}{\overset{*}{\sim}}, \overset{*}{\cap}_{\varepsilon})$ is a commutative, idempotent semiring without zero but with unity.

3.4.5.3. Corollary: $(S_Z(U), \underset{\cup}{\overset{*}{\sim}}, \overset{*}{\cap}_{\varepsilon})$ is a commutative, idempotent hemiring with unity.

3.4.5.4. Corollary: $(S_Z(U), \underset{\Delta}{\overset{*}{\sim}}, \overset{*}{\cap}_{\varepsilon})$ is a Boolean Ring and aslo a $(S_Z(U), \underset{\Delta}{\overset{*}{\sim}}, \overset{*}{\cap}_{\varepsilon})$ is a commutative, multiplicative idempotent hemiring with unity.

3.4.5.5. Corollary: $(S_Z(U), \emptyset_Z, U_Z, \overset{*}{\underset{\cup}{\sim}}, \overset{*}{\cap}_{\varepsilon})$ Boolean, De Morgan, Kleene and Stone algebra.

4. Conclusion

In this paper, a new soft set operation called, complementary extended intersection, is proposed and its algebraic properties, particularly by comparing with the intersection operation in classical set theory are investigated. We handle the distributions of complementary extended intersection over other different kinds of operations on soft sets. A thorough examination of the algebraic structures that the set of soft sets form with these operations is given, taking into account the distribution laws and the algebraic aspects of the soft set operations. We demonstrate how various significant algebraic structures, including semiring, near-semiring, hemiring, Boolean ring, Boolean Algebra, De Morgan Algebra, Kleene Algebra, and Stone Algebra, are formed by the collection of all soft sets with a fixed parameter together with complementary extended intersection and other types of soft sets. We hope this study contributes to the literature of not only soft set theory but also classical algebra, since studying the algebraic structures of soft sets in relation to novel soft set operations gives us a detailed understanding of their use as well as new examples of algebraic structures. Future research may examine various kinds of complementary extended soft set operations together with their distributions and characteristics to see what algebraic structures they generate in the classes of soft sets with a fixed parameter set.

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