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APPLYING FUZZY ANALYTIC HIERARCHY PROCESS (FAHP) α-CUT BASED AND TOPSIS METHODS TO DETERMINE REGENCIAL ROAD HANDLING PRIORITY (Case Study: Badung Regency - Bali)

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Abstract : This study examines regencial road handling priority using FAHP α-cut based and TOPSIS methods using Badung regency in Bali province as the case study area. The study is restricted to regencial roads under severe conditions. Adoption of these FAHP α-cut based and TOPSIS methods allow the researcher to have estimation regarding the overall road handling priority considering the decision makers's attitude from optimistic to pessimistic situations and decision makers's degree of confidence from under the most uncertain to certain comparisons. Meanwhile, the previous study has analysed the same case study area using FAHP extent analysis and TOPSIS methods with no consideration on decision makers's attitude and confidence. This study concluded that road handling determination using FAHP α cut based and TOPSIS methods, considering both pessimistic and moderate situations and highly optimistics situation with uncertain conditions of the decision makers, produced similar top prioty as it used FAHP extent analysis and TOPSIS methods.

Keywords: Road Handling, Fuzzy AHP α-cut based method, TOPSIS

Abstrak : Pada penelitian ini penentuan prioritas penanganan jalan di Kabupaten Badung, Bali dilakukan menggunakan metode FAHP α-cut based dan TOPSIS. Studi ini hanya menganalisis jalan-jalan kabupaten dengan kondisi rusak berat. Kedua metode FAHP α-cut based dan TOPSIS ini mampu untuk mengestimasi prioritas penanganan jalan dengan memperhitungkan prilaku dan tingkat kepercayaan para pengambil keputusan. Prilaku pengambil keputusan disini terkait dengan perasaan optimis/ pesimis sementara tingkat kepercayaan pengambil keputusan terkait dengan perbandingan situasi kepastian/ketidakpastian. Studi ini memperlihatkan bagaimana para pengambil keputusan di tingkat kabupaten dapat menilai dan memutuskan secara obyektif dan cepat kelemahan dan keunggulan dari setiap kriteria dan urutan prioritas penanganan ruas jalan untuk kondisi rusak berat. Analisis menggunakan metode FAHP α-cut based dan TOPSIS dalam situasi pesimis dan moderat serta situasi optimis dengan kondisi tidak pasti akan menghasilkan prioritas utama yang sama dengan menggunakan metode FAHP extent analysis and TOPSIS.

Kata-kata Kunci: Penanganan Jalan, Fuzzy AHP α-cut based method, TOPSIS

INTRODUCTION

In determining road handling priority includes the experts's subjective judgement which frequently yields vague relations between criteria and alternatives. In addition, the real decision process is usually come with some unclear and potential factors in practice, such as decision maker's degree of confidence and degree of optimism of decision making. The conventional Analytic Hierarchy Process (AHP) however, usually overlooking these kinds of factors so it can not be entirely put into practice. It is therefore, essential to set up the appropriate system to identify and find the relative importance of criteria for the determination of road handling priority.

In a previous study (Wedagama, 2010) Fuzzy Analytic Hierarchy Process (FAHP) extent analysis and TOPSIS methods were used to examine regencial road handling priority under severe conditions in Badung regency in Bali province. In addition, the study compared the results between FAHP and TOPSIS methods and the AHP and the SK. No. 77/KPTS/Db/1990 methods. The study concluded that FAHP extent analysis and TOPSIS methods were preferred to the AHP and the SK. No.77/KPTS/Db/1990 methods in determining Badung regencial road handling priority under severe conditions.

Fuzzy AHP in that study however, used fuzzy numbers for scoring road alternatives. As the result, the study have not considered some potential factors including the decision maker's degree of confidence (α) and degree of optimism of decision making (β). A different approach therefore, is required to reveal such factors. In this study, fuzzy numbers are therefore used to score judgments of evaluation criteria. In so doing, a crisp judgement matrix is incorporated with the index of optimism to deal with criteria weighting. More specifically, defuzzification is carried out by performing the interval performance matrix with α-cut and the optimism index (β).This is so called (FAHP) α-cut based method.

Similar to that previous study by Wedagama (2010), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is applied in this study to obtain the final ranking for each alternative. Using this technique, the best alternative would be the one that is closest to the positive ideal solution and furthest from the negative ideal solution (Ballı & Korukoğlu, 2009). The positive ideal solution would maximise the benefit criteria and minimise the cost criteria. On the other hand, the negative ideal solution maximises the cost criteria and minimises the benefit criteria. In other words, the positive ideal solution is constructed from all best values of realistic criteria, while negative ideal solution including all worst values of logical criteria (Wang & Elhag, 2006 in Dagdeviren, et.al, 2009).

This study aims to examine regencial road handling priority under severe conditions in Badung regency in Bali province as the case study area using Fuzzy Analytic Hierarchy Process (FAHP) α-cut based and TOPSIS methods. The same set of data used in a previous study (Wedagama, 2010) is employed in this study. FAHP α-cut based method is used to determine the weights of the criteria by experts and then TOPSIS method is used to determine road links handling priority. Further, changes of ideal solution under different risk environments are also simulated.

Fuzzy Numbers

As shown in Figure 1, fuzzy numbers are the specific categories of fuzzy quantities in which a fuzzy quantity M corresponding to a generalisation of a real number r. Logically, M(x) is employed as an indicator for measuring the closeness of M(x) predicting r. A fuzzy number M is a convex normalised fuzzy set that is usually expressed with a given real numbers interval between 0 and 1.

Triangular and trapezoidal fuzzy numbers are commonly used in practice. In fact, it is more common to use triangular fuzzy numbers (TFNs) since they are easy to compute. In addition, they are more practical to describe work processing in a fuzzy environment. A triangular fuzzy number, M is shown in Figure 1 (Ballı & Korukoğlu, 2009):

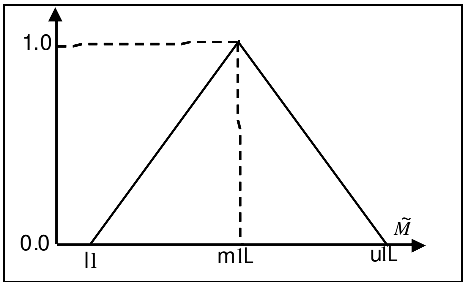


Figure 1 A Triangular Fuzzy Number, $\tilde{M}$

TFNs are expressed with three real numbers (l,m,u). The parameters l, m and u respectively, specify the smallest possible, the most promising and the largest possible values illustrating a fuzzy event. Their membership functions are defined as follows :

$$\mu(x / \tilde{M}) = \begin{cases} 0, & x < l, \\ (x-l)/(m-l), & l \le x \le m \\ (u-x)/(u-m), & m \le x \le u \\ 0, & x > u \end{cases}$$

Suppose the interval of confidence level is expressed with □, the triangular fuzzy number is defined as:

$\forall \alpha \in [0,1]$ and

$$\tilde{M}_\alpha = [l^\alpha, u^\alpha] = [(m-l)\alpha + \alpha, -(u-m)\alpha + u]$$
(2)

Some main operations for positive fuzzy numbers described by the interval of confidence are:

$$\begin{aligned} &\forall m_L, m_R, n_L, n_R \in R^+, \tilde{M}_\alpha = [m_L^\alpha, m_R^\alpha], \\ &\tilde{N}_\alpha = [n_L^\alpha, n_R^\alpha], \alpha \in [0,1] \\ &\tilde{M} \oplus \tilde{N} = [m_L^\alpha + n_L^\alpha, m_R^\alpha + n_R^\alpha] \\ &\tilde{M} \Theta \tilde{N} = [m_L^\alpha - n_L^\alpha, m_R^\alpha - n_R^\alpha] \qquad (3) \\ &\tilde{M} \otimes \tilde{N} = [m_L^\alpha n_L^\alpha, m_R^\alpha n_R^\alpha] \\ &\tilde{M} \div \tilde{N} = [m_L^\alpha / n_L^\alpha, m_R^\alpha / n_R^\alpha] \end{aligned}$$

The triangular fuzzy number, $\tilde{1}$ to $\tilde{9}$ are utilised to improve the conventional Saaty's nine-point scaling scheme. In order to take the imprecision of human qualitative assessments into consideration, the five triangular fuzzy numbers are defined with the corresponding membership functions as shown in Figure 2.

All elements in the judgement matrix and weight vectors are represented by triangular fuzzy number. The triangular fuzzy number for fuzzy judgement matrix decision process is defined as shown in Table 1.

Table 1. Fuzzy Number, Membership function and Linguistic Term

Fuzzy Number	Membership function	Linguistic Term
$\tilde{1}$	(1,1,3)	Very Poor
$\tilde{3}$	(1,3,5)	Poor
$\tilde{5}$	(3,5,7)	Ordinary
$\tilde{7}$	(5,7,9)	Excellent
$\tilde{9}$	(7,9,9)	Very Excellent

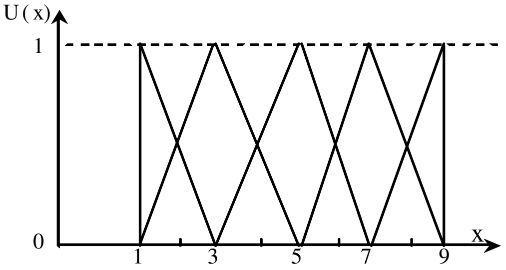


Figure 2. Triangular Fuzzy Ratio Scales

In comparison with FAHP extent analysis, FAHP α-cut-based method is less controversial because the uncertainty and the different attitude of decision maker are fully considered. On the other hand, the fuzzy extent analysis is easier in computation (Vahidnia, et.al, 2008).

FUZZY AHP AND TOPSIS

Fuzzy AHP

Each fuzzy number is corresponding to an interval value. A fuzzy ratio scale exactly represents a sub score ($\tilde{G}_{ijk}$) indicating the sub-score of alternative (A_i) with respect to sub-criterion (C_{jk}). After obtaining all sub-scores ($\tilde{G}_{ijk}$) of each alternatives (A_i) with respect to all sub-criteria, the judgement score ($\tilde{a}_{ij}$) is calculated. Equation (1) is used to separately aggregate all sub-scores of each alternative (A_i) with respect to the sub criteria (C_{jk}) which belong to the same criterion (C_j).

$$\tilde{G}_{ijk} = \sum_{k=1}^{q} \tilde{G}_{ijk}, \ i=1,2,\ldots.n$$

$$j=1,2,\ldots.m \qquad k=1,2,\ldots.q \qquad (1).$$

All scores from equation (1) are calculated to form a decision matrix as follows:

$$\begin{array}{c|cccc|} & C_1 & C_2 & \ldots.. & C_m \\ A_1 & \tilde{G}_{11} & \tilde{G}_{12} & \ldots.. & \tilde{G}_{1m} \\ A_2 & \tilde{G}_{21} & \tilde{G}_{22} & \ldots.. & \tilde{G}_{2m} \\ \ldots & \ldots.. & \ldots.. & \ldots.. & \ldots.. \\ \ldots. & \ldots.. & \ldots.. & \ldots\ldots & \ldots.. \\ A_n & \tilde{G}_{n1} & \tilde{G}_{n2} & \ldots\ldots & \tilde{G}_{nm} \end{array} \quad (2)$$

A normalisation process is conducted to allow a matching process with the weight vector. Each criterion (C_j) in a matrix above is normalised by using Equation (3). A fuzzy judgement matrix (A) is achieved as Equation (4) after normalising.

$$\tilde{a}_{ij} = \frac{\tilde{G}_{ij}}{\sqrt{\sum_{i=1}^{n} (\tilde{G}_{ij})^2}}, \text{ j = 1,2,....m} \quad (3)$$

$$A = \begin{array}{c|cccc|} & C_1 & C_2 & \ldots.. & C_m \\ A_1 & \tilde{a}_{11} & \tilde{a}_{12} & \ldots.. & \tilde{a}_{1m} \\ A_2 & \tilde{a}_{21} & \tilde{a}_{22} & \ldots.. & \tilde{a}_{2m} \\ \ldots & \ldots.. & \ldots.. & \ldots.. & \ldots.. \\ \ldots. & \ldots.. & \ldots.. & \ldots\ldots & \ldots.. \\ A_n & \tilde{a}_{n1} & \tilde{a}_{n2} & \ldots\ldots & \tilde{a}_{nm} \end{array} \quad (4)$$

Where $\tilde{a}_{ij}$ indicates the judgement score of alternative (A_j) with respect to criteria (C_j).

The fuzzy performance matrix represents the overall fuzzy performance which each alternative corresponds to all criteria. It is obtained by multiplying the fuzzy judgement matrix by the corresponding fuzzy weight vector.

Meanwhile, the weight vector represents the relative importance among each criterion is calculated with AHP pairwise comparison or with immediate expert's judgement. The different experts may define the different weight vectors because they usually produce the imprecise evaluation during the decision process. To handle this, a group of decisions on AHP with TFN is used to improve original pairwise comparison. A comprehensive pairwise comparison matrix (D) is constructed by integrating all decision makers' grades (b_{jep}) through equation (5)-(9). A score (b_{jep}) represents a decision maker (D_p) measures the relative importance by using Saaty's scale 1-9 between each criteria as shown in Table 2.

Table 2. Scale used for Pairwise Comparison (PC)

Intensity of Importance	Qualitative Definition	Explanation
1	Equally important	Two activities contribute equally to the objective
3	Moderately more important	Experience and judgements slightly favour one activity over another
5	Strongly more important	Experience and judgements strongly favour one activity over another
7	Very strongly more important	An activity is favoured very strongly over another and dominance is demonstrated in practice
9	Extremely more important	The evidence favouring activity over another is of the highest possible order of affirmation
2,4,6,8	Intermediate values	When compromise is needed
Reciprocals of the above numbers	If activity i has one of the above assigned to it when compared with activity j, then j has the reciprocal value when compared with with i.	

$$L_{je} = \min(b_{jep}), \text{ p=1,2,...t} \quad \text{j=1,2...m} \quad \text{e=1,2....m} \quad (5)$$

$$M_{je} = \frac{\sum_{p=1}^{t} b_{jep}}{p}, \text{ p=1,2,...t} \quad \text{j=1,2...m} \quad \text{e=1,2....m} \quad (6)$$

$$U_{je}=\max(U_{jep}),\ p=1,2,\ldots t\quad j=1,2\ldots m\quad e=1,2\ldots.m \quad (7)$$

$$\tilde{b}_{je} = (L_{je}, M_{je}, U_{je}),\quad j=1,2\ldots m\quad e=1,2\ldots.m \quad (8)$$

Where a comprehensive score ($\tilde{b}_{je}$) represents the relative importance among each criterion with triangular fuzzy numbers.The importance of each criterion is different. In order to acquire a weight ($\tilde{w}_j$) which corresponds to a specific criterion (C_j) the relative weights between all criteria is calculated as follows:

$$\tilde{w}_j = \frac{\sum_{e=1}^{m} \tilde{b}_{je}}{\sum_{j=1}^{m}\sum_{e=1}^{m} \tilde{b}_{je}},\quad j=1,2\ldots m\quad e=1,2\ldots.m \quad (9)$$

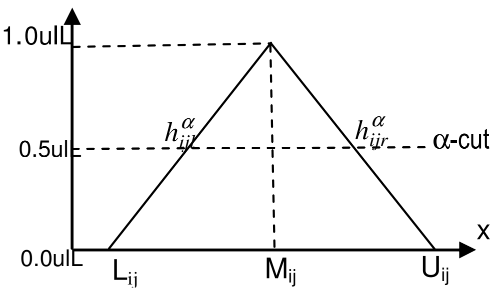
The criteria weights collectively make up a fuzzy weight vector (W) as in equation (10).

$$W = (\tilde{w}_1, \tilde{w}_2, \ldots\ldots\ldots\ldots. \tilde{w}_m) \quad (10)$$

The fuzzy performance matrix (H) is constructed by multiplying the fuzzy judgement matrix with the weight vector. This matrix represents the overall fuzzy performance scores of each alternative with respect to all criteria. The experts's subjective judgement produces uncertain and imprecise relations between criteria and alternatives. In addition, the real decision process is usually accompanied with some unclear and potential factors in practice, such as decision maker's degree of confidence and degree of optimism of decision making. The conventional AHP however, usually ignoring these kinds of factors so it can not be completely implemented in practical applications. To overcome this situation, defuzzification is carried out by performing the interval performance matrix with α-cut and the optimism index (β). The interval performance matrix (H_α) is computed by using α-cut method on the fuzzy performance matrix (H). Each fuzzy performance score ($\tilde{h}_{ij}$) is joined to respectively form an interval [$\tilde{h}^{\alpha}_{ijl}$, $\tilde{h}^{\alpha}_{ijr}$].

$$\tilde{h}^{\alpha}_{ijl} = L_{ij} + \alpha(M_{ij}-L_{ij}) \quad (11)$$

$$\tilde{h}^{\alpha}_{ijr} = U_{ij} + \alpha(U_{ij}-M_{ij}) \quad (12)$$



Where $\tilde{h}^{\alpha}_{ijl}$ and $\tilde{h}^{\alpha}_{ijr}$ respectively represent the left point and right point of the range of the triangle after using α-cut and the range of α is between 0 and 1. If the decision makers establish the higher degree of confidence (α), it shows they have asked sufficient information to support their decisions. Therefore, the higher degree of confidence is corresponding to the lower uncertainty.

The degree of optimism address the decision makers attitude that is may be optimistic, moderate or pessimistic. The optimism index is also applied to be a defuzzifier. Defuzzification is conducted by joining the optimism index to produce the final crisp numbers. The overall crisp performance matrix (H^{α}_{β}) is calculated as follows:

$$h^{\alpha}_{ij\beta} = \beta h^{\alpha}_{ijl} + (1-\beta)\, h^{\alpha}_{ijr},\quad 0 \le \alpha \le 1 \quad 0 \le \beta \le 1 \quad (13)$$

Where $h^{\alpha}_{ij\beta}$ indicates the crisp performance score which each alternative (Ai) corresponds to all criteria (C_j) under α degree of confidence and β degree of optimism.

Topsis

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method is one of the principal techniques for multicriteria decision making problems. TOPSIS defines two kinds of solutions consisting the positive ideal solution and the negative ideal solution. The positive ideal solution is regarded as the maximal benefits solution, and containing all best values of criteria. On the other hand, the negative ideal solution is treated as the minimal benefits solution and composed of the all worst values of criteria. TOPSIS defines solutions as the points which are nearest to the positive ideal point and farthest from the negative ideal solution at the same time.

The positive ideal solution ($h^{\alpha+}_{ij\beta}$) and the negative ideal solution ($h^{\alpha-}_{ij\beta}$) is determined as follows :

$$h^{\alpha+}_{j\beta} = \{(\max\ h^{\alpha}_{ij\beta} \mid j \in J), (\min\ h^{\alpha}_{ij\beta} \mid j \in J'), i = 1,2,...., n) \quad (14)$$

$$h^{\alpha-}_{j\beta} = \{(\min\ h^{\alpha}_{ij\beta} \mid j \in J), (\max\ h^{\alpha}_{ij\beta} \mid j \in J'), i = 1,2,...., n) \quad (15)$$

Where :

J={j=1,2,…m|j belongs to positive criteria}

J'={j=1,2,…m|j belongs to negative criteria}

After determining the ideal solution and negative ideal solution, the distance between positive ideal solution and negative ideal solution for each alternative is respectively calculated as follows :

$$s^{\alpha+}_{i\beta} = \sqrt{\sum_{j=1}^{m}(h^{\alpha}_{ij\beta} - h^{\alpha+}_{j\beta})^2} \quad ; i = 1,2,\ldots,n \quad (16)$$

$$s^{\alpha-}_{i\beta} = \sqrt{\sum_{j=1}^{m}(h^{\alpha}_{ij\beta} - h^{\alpha-}_{j\beta})^2} \quad ; i = 1,2,\ldots.,n \quad (17)$$

Where $s^{\alpha+}_{i\beta}$ and $s^{\alpha-}_{i\beta}$ represent the distance between the crisp performance scores of an alternative with respect to all criteria, all the positive and negative ideal solutions respectively. The relative closeness to the ideal solution for each alternative can be formulated using closeness coefficient (CC) as follows :

$$CC^{\alpha}_{i\beta} = \frac{s^{\alpha-}_{i\beta}}{s^{\alpha+}_{i\beta} + s^{\alpha-}_{i\beta}} \quad i = 1,2,\ldots n \quad (18)$$

Where $CC^{\alpha}_{i\beta}$ indicates a final performance score containing the decision maker's α degree of confidence about their valuations and degree of optimism. The larger final performance score expresses the more prior alternative.

MODEL APPLICATION AND RESULTS

Case Study Area and Data Descriptions

Figure 3 shows the location of Badung regency which is situated in the Southern Bali. This regency has a total roads lengths of 703.32 km (Statistics of Bali Province, 2008). Of these roadways, about 80% are regencial roads (552.17 km) while the rest including provincial roads and national roads. Of these regencial roads, 7, 41, 210 and 154 road links were under severe, damaged, moderate and good conditions respectively (Suyasa, 2008). This study, however focuses the analysis of road handling priority only to those road links under severe conditions as shown in Table 3.

Table 3. Road Links Description

Road Link Number	Descriptions
248	Pererenan – Padang Lenjong
400	Beringkit – Gegadon
153	Br. Pempatan Sembung – Balangan
90	Gerih – Latu
252	Balangan – Desa Sembung
165	Ungasan – Pura Massuka
353	Kantor Kades Cemagi - Kuburan

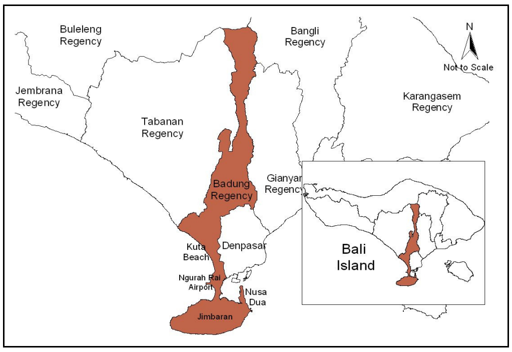


Figure 3. Case Study Area – Badung Regency

Analysis and Results

A hierarchical structure for road handling priority problem is shown in Figure 4. The ultimate goal, handling priority for regencial road under severe conditions, is located at level 1. At the next level, four major criteria are gathered so level 3 is consisted of sixteen sub-criteria. In addition, seven Badung regencial road link under severe conditions are locatede at the lowest level.

This problem is analysed from bottom to up. Each road link is measured by all sub-criteria to obtain sub-scores. Each criterion respectively sums up its sub-scores. Lastly, the more prior road link can be picked out.

Using the sama set od data in a previous study (Wedagama, 2010) the sub scores of each road link with respect to all sub-criteria are obtained as follows:

	A11	A12	A13	A14	A15	A16	B11	B12	B13	B14	B15	C11	C12	D11	D12	D13
248	$\tilde{1}$	$\tilde{5}$	$\tilde{1}$	$\tilde{5}$	$\tilde{5}$	$\tilde{1}$	$\tilde{9}$	$\tilde{9}$	$\tilde{3}$	$\tilde{9}$	$\tilde{1}$	$\tilde{3}$	$\tilde{9}$	$\tilde{9}$	$\tilde{9}$	$\tilde{9}$
400	$\tilde{1}$	$\tilde{5}$	$\tilde{1}$	$\tilde{5}$	$\tilde{5}$	$\tilde{1}$	$\tilde{9}$	$\tilde{9}$	$\tilde{5}$	$\tilde{9}$	$\tilde{3}$	$\tilde{3}$	$\tilde{7}$	$\tilde{9}$	$\tilde{9}$	$\tilde{9}$
153	$\tilde{5}$	$\tilde{1}$	$\tilde{1}$	$\tilde{5}$	$\tilde{1}$	$\tilde{1}$	$\tilde{9}$	$\tilde{9}$	$\tilde{3}$	$\tilde{9}$	$\tilde{1}$	$\tilde{3}$	$\tilde{3}$	$\tilde{9}$	$\tilde{1}$	$\tilde{1}$
90	$\tilde{1}$	$\tilde{5}$	$\tilde{1}$	$\tilde{5}$	$\tilde{5}$	$\tilde{1}$	$\tilde{9}$	$\tilde{9}$	$\tilde{5}$	$\tilde{9}$	$\tilde{3}$	$\tilde{3}$	$\tilde{5}$	$\tilde{9}$	$\tilde{9}$	$\tilde{1}$
252	$\tilde{5}$	$\tilde{1}$	$\tilde{1}$	$\tilde{5}$	$\tilde{1}$	$\tilde{1}$	$\tilde{9}$	$\tilde{9}$	$\tilde{5}$	$\tilde{9}$	$\tilde{3}$	$\tilde{3}$	$\tilde{7}$	$\tilde{9}$	$\tilde{9}$	$\tilde{9}$
165	$\tilde{3}$	$\tilde{5}$	$\tilde{1}$	$\tilde{5}$	$\tilde{5}$	$\tilde{1}$	$\tilde{9}$	$\tilde{9}$	$\tilde{7}$	$\tilde{9}$	$\tilde{5}$	$\tilde{1}$	$\tilde{3}$	$\tilde{9}$	$\tilde{1}$	$\tilde{1}$
353	$\tilde{3}$	$\tilde{5}$	$\tilde{1}$	$\tilde{5}$	$\tilde{5}$	$\tilde{1}$	$\tilde{9}$	$\tilde{9}$	$\tilde{9}$	$\tilde{9}$	$\tilde{9}$	$\tilde{1}$	$\tilde{9}$	$\tilde{9}$	$\tilde{9}$	$\tilde{1}$

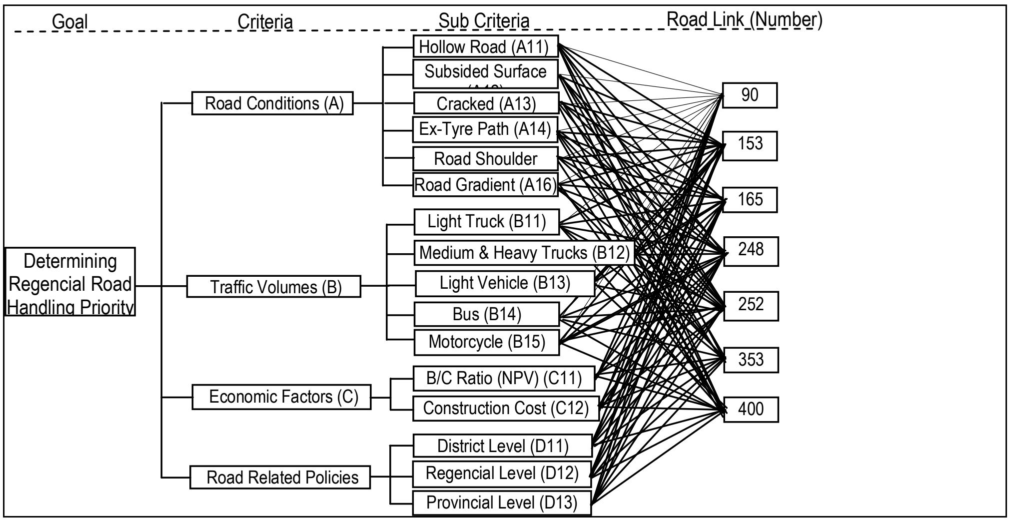


Figure 4. Hierarchy for Badung Regencial Road Handling Priority

By equation (1), all sub-scores of each road link are summed up with respect to the sub-criteria which belong to the same criterion to acquire all scores ($\tilde{G}_{ij}$). The scores of each road link with respect to road conditions (A) are calculated as:

$$\tilde{G}_{11} = \tilde{G}_{111} \oplus \tilde{G}_{112} \oplus \tilde{G}_{113} \oplus \tilde{G}_{114} \oplus \tilde{G}_{115} \oplus \tilde{G}_{116}$$

$$\tilde{G}_{11} = (1,1,3) \oplus (3,5,7) \oplus (1,1,3) \oplus (3,5,7) \oplus (3,5,7) \oplus (1,1,3) = (12,18,30)$$

The rest may be calculated using the same way, so that the G matrix can be formed as follows:

	A	B	C	D
248	(12, 18, 30)	(23, 31, 35)	(8, 12, 14)	(21, 27, 27)
400	(12, 18, 30)	(25, 35, 39)	(6, 10, 14)	(21, 27, 27)
153	(10, 14, 26)	(23, 31, 35)	(2, 6, 10)	(9, 11, 15)
90	(12, 18, 30)	(25, 35, 39)	(4, 8, 12)	(15, 19, 21)
252	(10, 14, 26)	(25, 35, 39)	(6, 10, 14)	(21, 27, 27)
165	(12, 20, 32)	(29, 39, 43)	(2, 4, 8)	(9, 11, 15)
353	(12, 20, 32)	(35, 45, 45)	(8, 10, 12)	(15, 19, 21)

A normalisation process is conducted using equation (3). Road conditions with respect to each road link is normalised as follows:

$$\tilde{a}_{11} = \frac{\tilde{G}_{11}}{\sqrt{\tilde{G}_{11}^2 \oplus \tilde{G}_{21}^2 \oplus \tilde{G}_{31}^2}} = \frac{(12,18,30)}{(30.332,46.519,78.102)} = (0.154,\ 0.387,\ 0.989)$$

Using the same way, the fuzzy judgement matrix (A) is constructed as follows:

	A	B	C	D
248	(0.154, 0.387, 0.989)	(0.220, 0.324, 0.495)	(0.248, 0.507, 0.935)	(0.354, 0.481, 0.614)
400	(0.154, 0.387, 0.989)	(0.240, 0.366, 0.552)	(0.186, 0.423, 0.935)	(0.354, 0.481, 0.614)
153	(0.128, 0.301, 0.857)	(0.220, 0.324, 0.495)	(0.062, 0.254, 0.668)	(0.152, 0.196, 0.341)
90	(0.154, 0.387, 0.989)	(0.240, 0.366, 0.552)	(0.124, 0.338, 0.802)	(0.253, 0.338, 0.477)
252	(0.128, 0.301, 0.857)	(0.240, 0.366, 0.552)	(0.186, 0.423, 0.935)	(0.354, 0.481, 0.614)
165	(0.154, 0.430, 1.055)	(0.278, 0.408, 0.608)	(0.062, 0.169, 0.535)	(0.152, 0.196, 0.341)
353	(0.154, 0.430, 1.055)	(0.335, 0.471, 0.636)	(0.248, 0.423, 0.802)	(0.253, 0.338, 0.477)

A comprehensive pairwise comparison matrix (D) is calculated by integrating the expert's different opinions using equation (5) – (8). The D matrix is obtained as follows:

	A	B	C	D
A	(1.000, 1.000,1.000)	(0.200, 1.931,4.000)	(0.143, 1.322,5.000)	(0.200, 2.198,4.000)
B	(0.250, 1.296,5.000)	(1.000, 1.000,1.000)	(0.200, 1.860,5.000)	(0.200, 2.681,5.000)
C	(0.200, 3.237,7.000)	(0.200, 1.860,5.000)	(1.000, 1.000,1.000)	(0.200, 2.623,5.000)
D	(0.250, 1.383,5.000)	(0.200, 1.083,5.000)	(0.200, 1.292,5.000)	(1.000, 1.000,1.000)

By using equation (9), the fuzzy weight vector (W) is obtained as follows:

WA = (0.024, 0.241, 2.173)
WB = (0.026, 0.255, 2.483)
WC = (0.025, 0.326, 2.794)
WD = (0.026, 0.178, 2.483)

The fuzzy weight vector and the fuzzy judgement matrix are then combined to construct fuzzy performance matrix. Each criterion weight is multiplied with its corresponding criterion in the fuzzy judgement matrix to obtain the fuzzy performance matrix (H) as follows:

	A	B	C	D
248	(0.004, 0.093, 2.149)	(0.006, 0.083, 1.229)	(0.006, 0.165, 2.613)	(0.009, 0.086, 1.524)
400	(0.004, 0.093, 2.149)	(0.006, 0.094, 1.370)	(0.005, 0.138, 2.613)	(0.009, 0.086, 1.524)
153	(0.003, 0.073, 1.863)	(0.006, 0.083, 1.229)	(0.002, 0.083, 1.867)	(0.004, 0.035, 0.847)
90	(0.004, 0.093, 2.149)	(0.006, 0.094, 1.370)	(0.003, 0.110, 2.240)	(0.007, 0.060, 1.186)
252	(0.003, 0.073, 1.863)	(0.006, 0.094, 1.370)	(0.005, 0.138, 2.613)	(0.009, 0.086, 1.524)
165	(0.004, 0.104, 2.292)	(0.007, 0.104, 1.510)	(0.002, 0.055, 1.493)	(0.004, 0.035, 0.847)
353	(0.004, 0.104, 2.292)	(0.009, 0.120, 1.581)	(0.006, 0.138, 2.240)	(0.007, 0.060, 1.186)

During the priority ranking process, some unobvious factors which usually are ignored may deeply affect the decision results. Therefore, the experts's degree of confidence and degree of optimism should be brought up during defuzzification process so that approaching the real decision.

The value of α indicates the experts' degree of confidence in their subjective evaluations concerning alternatives scores and criteria weight. The higher α value expresses the higher degree of confidence and closer to the possible value of the triangular fuzzy numbers. In addition, by using the β value (optimism index), defuzzification is conducted to obtain the crisp performance scores.

The crips performance scores and TOPSIS methods (equation 14-18) are employed to determine the road link priority. The results which also showing the sensitivity analyses are depicted in Figures 5-7. These graphs show β value as 0.05, 0.5 and 0.95 reflecting the pessimistic, the moderate and the optimistics situations respectively. In addition, the horizontal and vertical axes showing the α varying from 0 to 1 and the closeness coefficient (CC) values respectively.

If the real crisp number is overestimated (β>0.5), the value of $\tilde{a}_{ij}$ in the judgement matrix is higher than the central value. If it is underestimated ((β<0.5) the value of $\tilde{a}_{ij}$ is lower than the central value. Meanwhile, The CC indicates the distance of road links from positive ideal solution in which the higher CC value expressing the higher priority. From the graphs, mutual comparisons can be performed from the most uncertain situation (α=0) to the most uncertain situation (α=1), from which the relative Badung regencial road link handling priority can be realised.

Based on Figures 5 and 6, in both pessimistic and moderate situations (β=0.05 and β=0.50), the CC value of road links connecting between Pererenan and Padang Lenjong (248) and between Beringkit and Gegadon (400) are very close varying from the most uncertain situation (α = 0) to the most certain situations (α = 1).

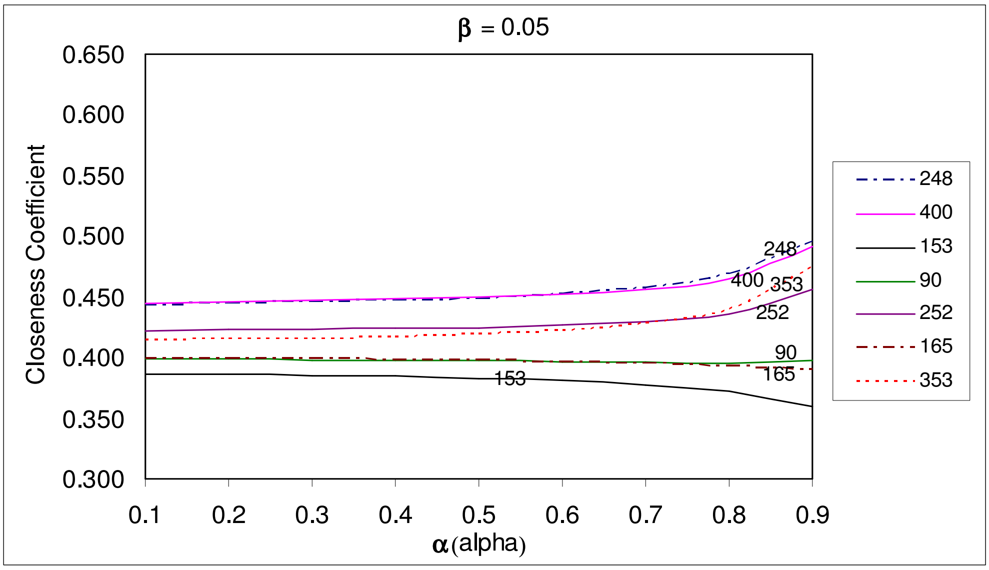


Figure 5. Sensitivity Analysis - Pessimistic Situation

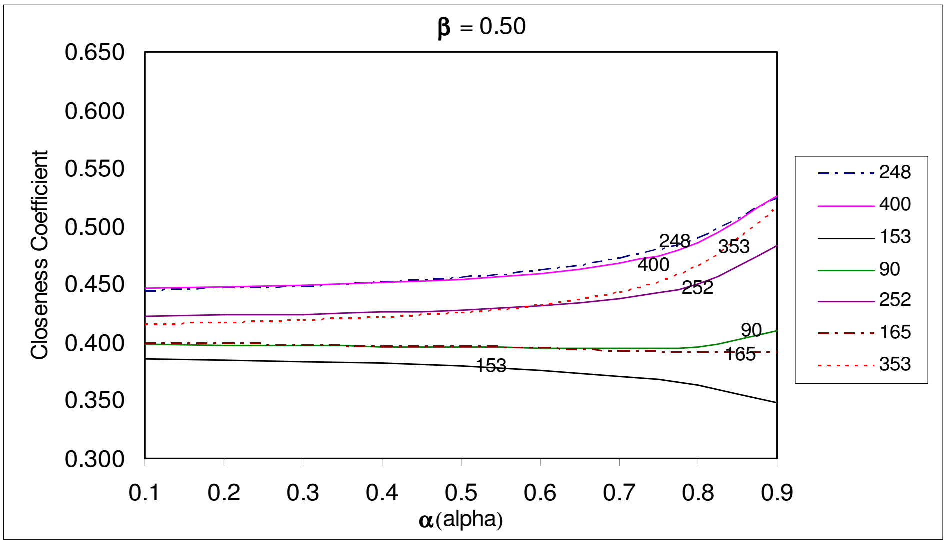


Figure 6. Sensitivity Analysis - Moderate Situation

For higly optimistic situation (β = 0.95), the CC value of road link connecting between Beringkit and Gegadon (400) is much higher than the one of road link connecting Pererenan and Padang Lenjong (248) under the most certain comparison (α = 0.9). However, the CC value of road link connecting Pererenan and Padang Lenjong (248) is slightly higher than the of road link connecting between Beringkit and Gegadon (400) under the most uncertain comparison (α = 0)

For all situations (pessimistic, moderate and optimistic) road link connecting between Br. Pempatan Sembung and Balangan (153) has the lowest CC value. This indicates that this road link has the lowest priority among the others for all situations ranging from under the most uncertain to certain comparisons.

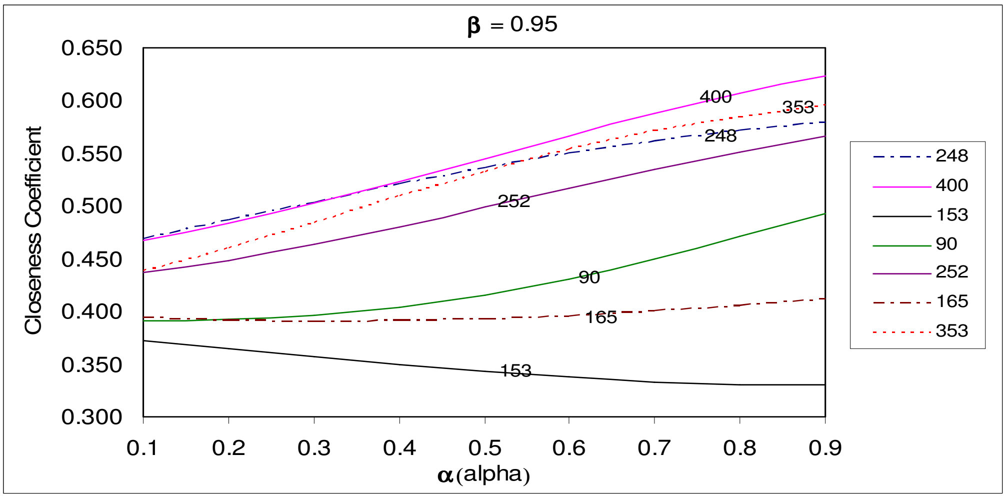


Figure 7. Sensitivity Analysis - Highly Optimistic Situation

Meanwhile, the ranking of road handling priority for the same set of road links from the previous study (Wedagama, 2010) is shown in Table 4. The table shows that road links 248 and 400 are the two regencial roads to have prominent road handling priority. As mentioned previously, this priority are based on model analyses using FAHP extent analysis and TOPSIS methods.

Table 4. Ranking of road links for road handling

No.	Road link Number	Road Link	Fuzzy AHP extent analysis & TOPSIS
1.	248	Pererenan – Padang Lenjong	1
2.	400	Beringkit – Gegadon	2
3.	153	Br. Pempatan Sembung – Balangan	3
4.	90	Gerih – Latu	4
5.	252	Balangan – Desa Sembung	5
6.	165	Ungasan – Pura Massuka	6
7.	353	Kantor Kades Cemagi - Kuburan	7

Source: Wedagama (2010)

In the mean time, considering both pessimistic and moderate situations (β=0.05 and β=0.50) in Figures 5 and 6, road links 248 and 400 are apparently the two regencial roads having first and second priorities respectively. In addition, determining Badung regencial road handling priority using FAHP α cut based and TOPSIS methods, under such decision makers's situations, produce the same top priority (i.e first and second priorities) as it used FAHP extent analysis and TOPSIS methods.

In contrast, under highly optimistic and gradually certain (α > 50%) situations, the decision makers's priorities are changed in comparison with the priorities resulted from using FAHP extent analysis and TOPSIS methods. In fact, during such optimistic and gradually certain circumstances, road link 400 is on top priority while road link 248 is on third priority.

Therefore, using FAHP α cut based and TOPSIS methods, under both pessimistic and moderate situations and highly optimistics situation with uncertain conditions of the decision makers, produced the same top prioty (i.e first and second priorities) as to determine road handling priorities using FAHP extent analysis and TOPSIS methods. In other words, using both FAHP α cut based and FAHP extent analysis with TOPSIS methods generally produce the same top priority for Badung regencial road links under severe circumstances.

Conclusions

In this study, Fuzzy AHP α-cut based and TOPSIS methods are employed to determine Badung regencial road handling priority. The study is restricted to roads under severe conditions. The decision maker's degree of optimism in Badung regency however, may have considerable impact on decision making. Adoption of these two methods allow the researcher to have estimation regarding the overall road handling priority from optimistic to pessimistic and from under the most uncertain to certain comparisons.

This study found that road handling determination using FAHP α cut based and TOPSIS methods, considering both pessimistic and moderate situations and highly optimistics situation with uncertain conditions of the decision makers, produced the same top prioty (i.e first and second priorities) as it used FAHP extent analysis and TOPSIS methods. In other words, using both FAHP α cut based and FAHP extent analysis with TOPSIS methods generally produce the same top priority for Badung regencial road links under severe circumstances.

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