E-Jurnal Matematika Vol. 9(1), Januari 2020, pp.96-103 ISSN: 2303-1751
DOI: https://doi.org/10.24843/MTK.2020.v09.i01.p285

OPTIMASI PENDISTRIBUSIAN PRODUK MENGGUNAKAN MODEL FUZZY MULTIOBJEKTIF CYCLICAL INVENTORY ROUTING PROBLEM

Eka Susanti[1§], Indrawati[2], Robinson Sitepu[3], Annisa Nabila[4], Riska Wulandari[5]

[1]Jurusan Matematika, Fakultas MIPA, Universitas Sriwijaya [Email: eka_susanti@mipa.unsri.ac.id]
[2]Jurusan Matematika, Fakultas MIPA, Universitas Sriwijaya [Email: iin10juni@yahoo.com]
[3]Jurusan Matematika, Fakultas MIPA, Universitas Sriwijaya [Email:robinson.sitepu@rocketmail.com]
[4]Jurusan Matematika, Fakultas MIPA, Universitas Sriwijaya [aniisanbl@gmail.com]
[5]Jurusan Matematika, Fakultas MIPA, Universitas Sriwijaya [riskawlndr07@gmail.com]
[§]*Corresponding Author*

ABSTRACT

In distribution activities, travel time is influenced by several factors, such as vehicle conditions, traffic conditions and road conditions. Uncertain traffic conditions cause the travel time can not be defined with certainty (uncertain). If several variables are in a uncertain condition then a deterministic approach is not appropriate. Fuzzy approach, can be used to overcome uncertainty conditions. In this study, the optimal route for chicken egg delivery using the fuzzy Multiobjective Cyclical Inventory Routing Problem (FMOCIRP) model with time and cost is expressed by triangular fuzzy numbers. Model completion using Winqsb software. An optimal shipping route is obtained with a total cost of Rp 114,000 and a total delivery time of 132 minutes. The first optimal delivery route is distributor, retailer 1, retailer 2, retailer 3 and back to the distributor. The second route is distributor, retailer 5, retailer 4 and back to the distributor.

Keywords*: Optimization, Fuzzy, Multiobjective*

1. PENDAHULUAN

Inventory Routing Problem (IRP) adalah permasalahan *inventory* dimana *supplier* bertanggung jawab untuk mengkoordinasi penambahan persediaan pengecer dan rute kendaraan dalam proses pendistribusian. IRP merupakan penggabungan antara beberapa komponen yaitu *inventory*/persediaan dan perutean kendaraan yang akan dicapai secara simultan. Penelitian terkait permasalahan IRP telah banyak dilakukan dan diaplikasikan ke berbagai bidang. Permasalahan multi depot IRP dan penyelesaian model IRP dengan metode *Branch and Cut* telah dibahas oleh (Guimaraes et al. 2019). Permasalahan IRP dengan multi depot juga telah dibahas oleh (Bertazzi et al. 2019). Permasalahan IRP dengan kendala dalam kondisi deterministik telah dibahas oleh (Christos, Jose, and Pinto 2017). Permasalahan IRP untuk distribusi produk yang *perishable* (mudah rusak) telah dibahas oleh (Azadeh et al. 2017) dan (Hu, Toriello, and Dessouky 2018). Permasalahan IRP juga dikembangkan oleh (Chitsaz, Divsalar, and Vansteenwegen 2016) untuk pola distribusi *cyclic*, selanjutnya disebut model *Cyclical Inventory Routing Problem* (CIRP). Permasalahan IRP dengan *multi vehicle* diperkenalkan oleh (Santos, Satoru, and Simonetti 2016) dan (Soysal 2016).

Beberapa penelitian sebelumnya membahas permasalahan IRP dan CIRP dengan satu fungsi tujuan yaitu meminimumkan biaya distribusi. Pada beberapa kasus, permasalahan IRP menginginkan lebih dari satu tujuan. Model *Multiobjective Green Cyclical Inventory Routing Problem* (MOGCIRP) dengan mempertimbangkan biaya dan total emisi kendaraan sebagai fungsi tujuan diperkenalkan oleh (Rau, Daniel, and Agus 2018). Beberapa metode untuk menyelesaikan permasalahan multiobjektif telah banyak dikembangkan, diantaranya (Liang et al. 2018) memperkenalkan algoritma *hybrid bat* dengan mempertimbangkan faktor ekonomi dan emisi.

Pada kegiatan distribusi, waktu tempuh dipengaruhi oleh beberapa faktor, seperti kondisi kendaraan, kondisi lalu lintas dan kondisi jalan. Kondisi lalu lintas yang tidak menentu mengakibatkan waktu tempuh tidak dapat didefinisikan dengan pasti (*uncertain*). Jika beberapa variabel dalam model dalam kondisi *uncertain* maka pendekatan deterministik tidak tepat digunakan. Pendekatan stokastik, pendekatan *fuzzy*, pendekatan *Robust* dan pendekatan probabilistik dapat digunakan untuk mengatasi kondisi *uncertainty*. Berikut diberikan beberapa penelitian terkait kondisi *uncertainty*. Optimasi stokastik untuk permasalahan *inventory perishable product* dibahas oleh (Azadi et al. 2019), (Hu et al. 2019) menggunakan pendekatan *fuzzy* untuk permasalahan multiobjektif, (Liu and Dong 2018) menggunakan pendekatan *robust* untuk permasalahan *multiobjective thermal system power plant*, (Rahimi, Baboli, and Rekik 2017) menggunakan pendekatan stokastik untuk permasalahan multiobjektif, (Nikzad, Bashiri, and Oliveira 2018) membahas pendekatan stokastik pada permasalahan distribusi obat menggunakan model IRP *under uncertainty*, (Palacios et al. 2017) menggunakan pendekatan *robust* untuk menyelesaikan permasalahan multiobjektif pada penyusunan jadwal pekerja took, (Susanti, Dwipurwani, and Yuliza 2017) menggunakan pendekatan *fuzzy* pada permasalahan pengangkutan sampah. Penelitian ini mengimplementasikan model FMOCIRP pada distribusi telur ayam dengan biaya dan waktu dinyatakan dengan bilangan *fuzzy*. Bilangan *fuzzy* yang digunakan adalah *Triangular Fuzzy Number* (TFN). Model FMOCIRP merupakan pengembangan model IRP dan permasalahan multiobjektif yang telah dibahas pada penelitian (Rau et al. 2018). Akan tetapi pada penelitian ini fungsi tujuan yaitu fungsi waktu dan fungsi biaya dinyatakan dengan TFN. Berikut diberikan model FMOCIRP dengan dua fungsi tujuan.

Diberikan fungsi tujuan

$$\min Z_1 = c\, x_{klij} \text{ (Fungsi biaya)}$$
$$\min Z_2 = e\, x_{klij} \text{ (Fungsi total emisi)}$$

Fungsi kendala : (1)

$\sum_{k\in O}\sum_{l\in p_k}\sum_{i\in p_{ki}} x_{klij} = 1; \quad \forall\, j, j \in S$

$x_{klcj} \le 1; \quad \forall\{k,l,j\}, j \in S.\, l \in p_{k,} k \in O$

$x_{klic} \le 1; \quad \forall\{k,l,i\}, i \in S.\, l \in p_{k,} k \in O$

$\sum_{k\in O}\sum_{l\in p_k}\sum_{i\in p_{ki}} x_{klij} - \sum_{k\in O}\sum_{l\in p_k}\sum_{m\in p_{kl}} x_{kljm} = 0; \forall\, j, j \in S$

$\sum_{k\in O}\sum_{(l\in p_{kl}\in p_{k)}} z_{kij} - \sum_{k\in O}\sum_{(l\in p_{kl}\in p_{k)}} z_{kjl} = q_j^2 \times \sum_{k\in O}\sum_{l\in p_k}\sum_{i\in p_{kl}} x_{klij}\,; \forall\, j \in S$

$T_k^{min} \le ct_k \le T_k^{max}; \forall\, k \in O$

$q_{kl}^1 = \sum_{j\in p_{ki}} z_{kcj}; \forall\,\{k,l\}, k \in O, l \in p_k$

$\{x_{klij}, y_k\} \in \{0,1\}; \forall\{k,l,i,j\},$ $\{i,j\} \in S, k \in O, l \in p_k$

$\{ct_k, z_{kij}\} \ge 0; \forall\{k,i,j\}, \{i,j\} \in S,$ $k \in O, l \in p_k$

dengan

x_{klij} variabel keputusan rute dari titik *i* ke titik *j*

c pusat distribusi

S himpunan titik sales

O *cyclic tour* , $O = \{p_1 \cdots p_0 \cdots p_k\}$

p_k *single tour*

$p_k = \{c, s_a, s_b, \cdots, s_i, c\}$, $s_i \in S$, atau $p_k = \{p_{k1} \cdots p_{kl}\}$

$p_{kl} = \{c, s_a, s_b, \cdots, s_i, c\}$, $s_i \in S \subset p_{kl}$

q_j^2 kuantitas penambahan produk untuk setiap titik *j*

ct_k waktu siklus

T_k^{min} minimum waktu siklus untuk setiap *k*

T_k^{max} maksimum waktu siklus untuk setiap *k*

q_{kl}^1 kuantitas produk yang diangkut pada *subtour l* di *cyclic tour k*

z_{kij} tingkat produk yang diangkut oleh kendaraan *k* ketika perjalanan dari titik *i* ke titik *j*

D_{ij} Jarak titik *i* ke titik *j*

AS kecepatan rata-rata kendaraan

DR_i tingkat permintaan titik *i*

UT Biaya pengiriman rata-rata

VC kapasitas maksimum kendaraan

$T_k^{min} = \sum_{i\in p_k}\sum_{j\in p_k}\left(D_{ij}/AS\right)/\left(1 - 2\sum_{i\in p_k} DR_i/UT\right)$

$T_{kl}^{max} = VC/\sum_{i\in p_{kl}\in p_k} DR_i$

$T_k^{max} = min\{T_{k1}^{max} \cdots T_{kl}^{max}\}$

2. METODE PENELITIAN

Berikut diberikan langkah-langkah penyelesaian permasalahan optimasi pendistribusian menggunakan model FMOCIRP

1. Pengumpulan Data.
   Data yang digunakan dalam penelitian ini adalah data primer. Terdiri dari data jumlah pengecer, banyak permintaan rata-

E-Jurnal Matematika Vol. 9(1), Januari 2020, pp.96-103 ISSN: 2303-1751
DOI: https://doi.org/10.24843/MTK.2020.v09.i01.p285

rata pengecer perhari, biaya dan waktu pengiriman.

2. Pengembangan Model *Fuzzy Multiobjektive Inventory Routing Problem* (FMOCIRP). Model FMOCIRP merupakan pengembangan model yang MOGCIRP yang diperkenalkan oleh (Rau et al. 2018). Pada (Rau et al. 2018) nilai parameter diketahui dengan pasti atau bersifat deterministik. Pada penelitian ini parameter biaya dan waktu dipertimbangkan dalam kondisi ketidakpastian dan dinyatakan dalam bentuk bilangan *fuzzy*. Berikut diberikan fungsi tujuan model FMOCIRP.

   Diberikan fungsi tujuan

   $\min Z_1 = \tilde{c}\, x_{klij}$ (Fungsi biaya) (2)
   $\min Z_2 = \tilde{t}\, x_{klij}$ (Fungsi waktu)
   Kendala model FMOCIRP yang digunakan analog dengan model MOGCIRP

3. Mentransformasi model FMOCIRP (2) ke bentuk deterministik
   Langkah-langkah penyelesaian adalah sebagai berikut
   a. Menentukan penyelesaian *single* objektif Z_1 dan Z_2 masing-masing diperoleh solusi $\{X_1^1, X_1^2\}$ dan $\{X_2^1, X_2^2\}$
   b. Menentukan nilai target untuk Z_1 dan Z_2, yaitu
   $L_k = min\{z_k^1(X_1^1), z_k^2(X_1^2), z_k^1(X_1^1), z_k^2(X_1^2)\}, k = 1,2$
   $U_k = max\{z_k^1(X_1^1), z_k^2(X_1^2), z_k^1(X_1^1), z_k^2(X_1^2)\}, k = 1,2$

Diperoleh bentuk deterministik model (2), yaitu

$maks\ \lambda$

Kendala

$$\lambda \le \frac{Z_1(X) - U_1}{L_1 - U_1} \qquad (3)$$
$$\lambda \le \frac{Z_2(X) - U_2}{L_2 - U_2}$$

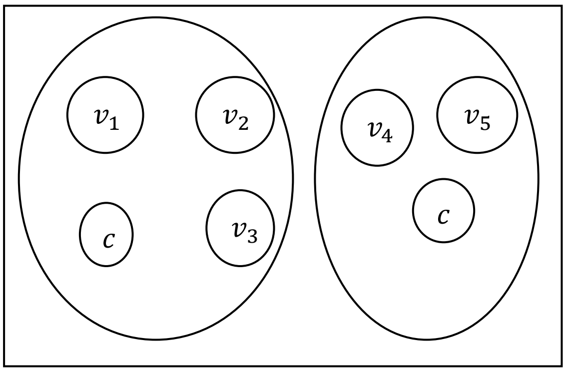
$\sum_{k \in O} \sum_{l \in p_k} \sum_{i \in p_{ki}} x_{klij} = 1; \quad \forall\, j, j \in S$
$x_{klcj} \le 1; \quad \forall \{k, l, j\}, j \in S, l \in p_{k,} k \in O$
$x_{klic} \le 1; \quad \forall \{k, l, i\}, i \in S, l \in p_{k,} k \in O$
$\sum_{k \in O} \sum_{l \in p_k} \sum_{i \in p_{ki}} x_{klij} - \sum_{k \in O} \sum_{l \in p_k} \sum_{m \in p_{kl}} x_{kljm} = 0; \forall\, j, j \in S$
$\sum_{k \in O} \sum_{(l \in p_{kl} \in p_k)} z_{kij} - \sum_{k \in O} \sum_{(l \in p_{kl} \in p_k)} z_{kjl} = q_j^2 \times \sum_{k \in O} \sum_{l \in p_k} \sum_{i \in p_{kl}} x_{klij}; \forall\, j \in S$
$T_k^{min} \le ct_k \le T_k^{max}; \forall\, k \in O$
$q_{kl}^1 = \sum_{j \in p_{ki}} z_{kcj}; \forall \{k, l\}, k \in O, l \in p_k$
$\{x_{klij}, y_k\} \in; \forall \{k, l, i, j\}, \{i, j\} \in S,$
$k \in O, l \in p_k$ $\{ct_k, z_{kij}\} \ge 0; \forall \{k, i, j\}, \{i, j\} \in S,$
$k \in O, l \in p_k$

Penyelesaian Model (3) menggunakan software Winqsb.

3. HASIL DAN PEMBAHASAN

Pada makalah ini ditentukan rute optimal pengiriman telur ayam dari distributor ke lima pengecer dengan mempertimbangkan biaya dan waktu pengiriman, permintaan pengecer dan kapasitas maksimal kendaraan. Fungsi tujuan Z_1 adalah fungsi yang meminimunkan total biaya pengiriman. Koefisien dari variabel x_{klij} adalah TFN untuk biaya pengiriman dari titik *i* ke titik *j*. Fungsi tujuan Z_2 adalah fungsi yang meminimumkan total waktu pengiriman. Waktu pengiriman terdiri dari waktu tempuh dan waktu *loading*. Biaya pengiriman rata-rata persiklus tour (UT) sebesar Rp.59.000, kecepatan rata kendaraan 40 km/jam. Permintaan rata-rata di titik 1 sampai dengan 5 berturut-turut sebagai berikut, 60 kg, 60 kg, 45 kg, 90 kg, 45 kg. Bahan bakar yang digunakan kendaraan adalah bensin. Biaya dalam satuan rupiah dan waktu dengan satuan menit. Diasumsikan bahwa rute dapat dilalui bolak balik atau jalur dua arah. Kapasitas maksimal kendaraan adalah 170 kg.

Mempertimbangkan kapasitas maksimal kendaraan, dibentuk dua wilayah pengiriman, yaitu tour 1 dan tour 2 $(l = 1,2)$ menggunakan satu jenis kendaraan $(k = 1)$ atau dapat dituliskan $p_{kl} = \{p_{11}, p_{12}\}$, dimana $p_{11} = \{c, v_1, v_2, v_3, c\}$, $p_{12} = \{c, v_4, v_5, c\}$. Titik c adalah pusat distribusi, selanjutnya disebut depot. Pembagian wilayah pengiriman diberikan pada Gambar 1 berikut.



Gambar 1. Pembagian Wilayah Pengiriman

Menggunakan model FMOCIRP, ditentukan rute pengiriman berdasarkan pembagian wilayah pada Gambar 1. Berikut diberikan formulasi model FMOCIRP.

$Min\ Z_1 =$
$(20000; \widetilde{20000}; 25000)x_{11cv1} +$
$(20000; \widetilde{20000}; 25000)x_{12cv1} +$
$(4000; \widetilde{4000}; 9000)\ x_{11v2v1} +$
$(4000; \widetilde{4000}; 9000)x_{12v2v1} +$
$(8500; \widetilde{8500}; 13500)\ x_{11v3v1} +$
$(8500; \widetilde{8500}; 13500)x_{12v3v1} +$
$(25000; \widetilde{25000}; 30000)\ x_{11cv2} +$
$(25000; \widetilde{25000}; 30000)x_{12cv2} +$
$(4000; \widetilde{4000}; 90000)x_{11v1v2} +$
$(4000; \widetilde{4000}; 9000)x_{12v1v2} +$
$(4500; \widetilde{4500}; 9500)x_{11v3v2} +$
$(4500; \widetilde{4500}; 9500)x_{12v3v2} +$
$(28500; \widetilde{28500}; 33500)x_{11cv3} +$
$(28500; \widetilde{28500}; 33500)x_{12cv3} +$
$(8500; \widetilde{8500}; 13500)x_{11v1v3} +$
$(8500; 8500; 13500)x_{12v1v3} +$
$(4500; \widetilde{4500}; 9500)x_{11v2v3} +$
$(4500; \widetilde{4500}; 9500)x_{12v2v3} +$
$(20000; \widetilde{20000}; 25000)x_{11v1c} +$
$(20000; \widetilde{20000}; 25000)x_{11v1c} +$
$(25000; \widetilde{25000}; 30000)x_{11v2c} +$
$(25000; \widetilde{25000}; 30000)x_{12v2c} +$
$(28500; \widetilde{28500}; 33500)x_{11v3c} +$
$(28500; \widetilde{28500}; 33500)x_{12v3c} +$
$(26000; \widetilde{26000}; 31000)x_{11cv4} +$
$(26000; \widetilde{26000}; 31000)x_{12cv4} +$
$(1000; \widetilde{1000}; 6000)x_{11v5v4} +$
$(1000; \widetilde{1000}; 6000)x_{12v5v4} +$
$(30000; \widetilde{30000}; 35000)x_{11cv5} +$
$(30000; \widetilde{30000}; 35000)x_{12cv5} +$
$(1000; \widetilde{1000}; 6000)x_{11v4v5} +$
$(1000; \widetilde{1000}; 6000)x_{12v4v5} +$
$(26000; \widetilde{26000}; 31000)x_{11v4c} +$
$(26000; \widetilde{26000}; 31000)x_{12v4c} +$
$(30000; \widetilde{30000}; 35000)x_{11v5c} +$
$(30000; \widetilde{30000}; 35000)$ (4)

$Min\ Z_2 = (20; \widetilde{20}; 25)x_{11cv1} +$
$(20; \widetilde{20}; 25)x_{12cv1} + (8; \widetilde{8}; 13)\ x_{11v2v1} +$
$(8; \widetilde{8}; 13)\ x_{12v2v1} + (15; \widetilde{15}; 20)\ x_{11v3v1} +$
$(15; \widetilde{15}; 20)\ x_{12v3v1} + (24; \widetilde{24}; 29)\ x_{11cv2} +$
$(24; \widetilde{24}; 29)x_{12cv2} + (8; \widetilde{8}; 13)x_{11v1v2} +$
$(8; \widetilde{8}; 13)x_{12v1v2} + (9; \widetilde{9}; 14)x_{11v3v2} +$
$(9; \widetilde{9}; 14)x_{12v3v2} + (37; \widetilde{37}; 42)x_{11cv3} +$
$(37; \widetilde{37}; 42)x_{12cv3} + (15; \widetilde{15}; 20)\ x_{11v1v3} +$
$(15; \widetilde{15}; 20)\ x_{12v1v3} + (9; \widetilde{9}; 14)x_{11v2v3} +$
$(9; \widetilde{9}; 14)x_{12v2v3} + (20; \widetilde{20}; 25)x_{11v1c} +$
$(20; \widetilde{20}; 25)x_{11v1c} + (24; \widetilde{24}; 29)x_{11v2c} +$
$(24; \widetilde{24}; 29)x_{12v2c} + (37; \widetilde{37}; 42)x_{11v3c} +$
$(37; \widetilde{37}; 42)x_{12v3c} + (26; \widetilde{26}; 31)x_{11cv4} +$
$(26; \widetilde{26}; 31)x_{12cv4} + (4; \widetilde{4}; 9)x_{11v5v4} +$
$(4; \widetilde{4}; 9)x_{12v5v4} + (28; \widetilde{28}; 33)x_{11cv5} +$
$(28; \widetilde{28}; 33)x_{12cv5} + (4; \widetilde{4}; 9)x_{11v4v5} +$
$(4; \widetilde{4}; 9)x_{12v4v5} + (26; \widetilde{26}; 31)x_{11v4c} +$
$(26; \widetilde{26}; 31)x_{12v4c} + x_{12cv4} +$
$(4; \widetilde{4}; 9)x_{11v5v4} + (4; \widetilde{4}; 9)x_{12v5v4} +$
$(28; \widetilde{28}; 33)x_{11cv5} + x_{11v5c} + x_{12cv4} +$
$(4; \widetilde{4}; 9)x_{11v5v4} + (4; \widetilde{4}; 9)x_{12v5v4} +$
$(28; \widetilde{28}; 33)x_{11cv5} + x_{12v5c}$

Kendala

$x_{11cv1} + x_{12cv1} + x_{11v2v1} + x_{12v2v1} + x_{11v3v1} + x_{12v3v1} = 1$
$x_{11cv2} + x_{12cv2} + x_{11v1v2} + x_{12v1v2} + x_{11v3v2} + x_{12v3v2} = 1$
$x_{11cv3} + x_{12cv3} + x_{11v1v3} + x_{12v1v3} + x_{11v2v3} + x_{12v2v3} = 1$
$x_{11cv4} + x_{12cv4} + x_{11v5v4} + x_{12v5v4} = 1$
$x_{11cv5} + x_{12cv5} + x_{11v4v5} + x_{12v4v5} = 1$
$x_{11cv1} + x_{12cv1} + x_{11cv2} + x_{12cv2} + x_{11cv3} + x_{12cv3} = 1$

E-Jurnal Matematika Vol. 9(1), Januari 2020, pp.96-103 ISSN: 2303-1751
DOI: https://doi.org/10.24843/MTK.2020.v09.i01.p285

$x_{11v2v1} + x_{12v2v1} + x_{11v1v2} + x_{12v1v2} + x_{11v1v3} + x_{12v1v3} = 1$
$x_{11v3v1} + x_{12v3v1} + x_{11v3v2} + x_{12v3v2} + x_{11v2v3} + x_{12v2v3} = 1$
$x_{11cv4} + x_{12cv4} + x_{11cv5} + x_{12cv5} = 1$
$x_{11v5v4} + x_{12v5v4} + x_{11v4v5} + x_{12v4v5} = 1$
$x_{11cv1} \leq 1$; $x_{12cv1} \leq 1$; $x_{11cv2} \leq 1$; $x_{12cv2} \leq 1$; $x_{11cv3} \leq 1$; $x_{12cv3} \leq 1$
$x_{11cv4} \leq 1$; $x_{12cv4} \leq 1$; $x_{11cv5} \leq 1$; $x_{12cv5} \leq 1$
$x_{11v1c} \leq 1$; $x_{12v1c} \leq 1$; $x_{11v2c} \leq 1$; $x_{12v2c} \leq 1$; $x_{11v3c} \leq 1$; $x_{12v3c} \leq 1$
$x_{11v4c} \leq 1$; $x_{12v4c} \leq 1$; $x_{11v5c} \leq 1$; $x_{12v5c} \leq 1$
$x_{11cv1} + x_{12cv1} + x_{11v2v1} + x_{12v2v1} + x_{11v3v1} + x_{12v3v1} - x_{11v1c} - x_{12v1c} - x_{11v1v2} - x_{12v1v2} - x_{11v1v3} - x_{12v1v3} = 0$
$x_{11cv2} + x_{12cv2} + x_{11v1v2} + x_{12v1v2} + x_{11v3v2} + x_{12v3v2} - x_{11v2c} - x_{12v2c} - x_{11v2v1} - x_{12v2v1} - x_{11v2v3} - x_{12v2v3} = 0$
$x_{11cv3} + x_{12cv3} + x_{11v1v3} + x_{12v1v3} + x_{11v2v3} + x_{12v2v3} - x_{11v3c} - x_{12v3c} - x_{11v3v1} - x_{12v3v1} - x_{11v3v2} - x_{12v3v2} = 0$
$x_{11cv3} + x_{12cv3} + x_{11v1v3} + x_{12v1v3} + x_{11v2v3} + x_{12v2v3} - x_{11v3c} - x_{12v3c} - x_{11v3v1} - x_{12v3v1} - x_{11v3v2} - x_{12v3v2} = 0$
$x_{11cv4} + x_{12cv4} + x_{11v5v4} + x_{12v5v4} - x_{11v4c} - x_{12v4c} - x_{11v4v5} - x_{12v4v5} = 0$
$x_{11cv5} + x_{12cv5} + x_{11v4v5} + x_{12v4v5} - x_{11v5c} - x_{12v5c} - x_{11v5v4} - x_{12v5v4} = 0$
$z_{1cv1} + z_{1v2v1} + z_{1v3v1} - z_{1v1c} - z_{1v1v2} - z_{1v1v3} = (60)^2(x_{11cv1} + x_{12cv1} + x_{11v2v1} + x_{12v2v1} + x_{11v3v1} + x_{12v3v1})$
$z_{1cv2} + z_{1v1v2} + z_{1v3v2} - z_{1v2c} - z_{1v2v1} - z_{1v2v3} = (60)^2(x_{11cv2} + x_{12cv2} + x_{11v1v2} + x_{12v1v2} + x_{11v3v2} + x_{12v3v2})$
$z_{1cv3} + z_{1v1v3} + z_{1v2v3} - z_{1v3c} - z_{1v3v1} - z_{1v3v2} = (45)^2(x_{11cv3} + x_{12cv3} + x_{11v1v3} + x_{12v1v3} + x_{11v2v3} + x_{12v2v3})$
$z_{1cv4} + z_{1v5v4} - z_{1v4c} - z_{1v4v5} = (90)^2(x_{11cv4} + x_{12cv4} + x_{11v5v4} + x_{12v5v4})$
$z_{1cv5} + z_{1v4v5} - z_{1v5c} - z_{1v5v4} = (45)^2(x_{11cv5} + x_{12cv5} + x_{11v4v5} + x_{12v4v5})$
$180 \leq ct_1 \leq 222$
$z_{1cv1} + z_{1cv2} + z_{1cv3} \leq 9225$
$z_{1cv4} + z_{1cv5} \leq 10125$
$x_{klij} \in \{0,1\}$; $z_{kij}, ct_1 \geq 0$

Penyelesaian model (4) menggunakan software Winqsb, diperoleh solusi *single* objektif untuk masing-masing nilai TFN adalah $Z_1(X_1^1) = 114000$, $Z_1(X_1^2) = 149000$, $Z_1(X_2^1) = 115, Z_1(X_2^2) = 150$ dengan $X_1^1 = X_1^2 = \{x_{11v2v1} = 1; x_{11v3v2} = 1; x_{11cv3} = 1; x_{11v1c} = 1; x_{11v4v5} = 1; x_{11cv5} = 1; x_{11v4c};\ z_{1cv1} = 3600; z_{1cv2} = 3600; z_{1cv3} = 2025; z_{1cv4} = 8100;\ z_{1cv5} = 2025\}$. Nilai $Z_2(X_2^1) = 126, Z_2(X_2^2) = 161, Z_2(X_1^1) = 132$, $Z_2(X_1^2) =$167 dengan $X_2^1 = X_2^2 = \{x_{11cv1} = 1; x_{11v3v2} = 1;\ x_{11v1v3} = 1; x_{11v2c} = 1; x_{11v5v4} = 1; x_{11cv5} = 1; x_{11v4c} = 1\ ; z_{1cv1} = 3600; z_{1cv2} = 3600; z_{1cv3} = 2025; z_{1cv4} = 8100;\ z_{1cv5} = 2025\}$. Nilai $U_1 = 150000$; $L_1 = 114000$; $U_1 = 126$; $L_1 = 167$. Dapat dibentuk model deterministik dari Permasalahan (4) adalah sebagai berikut.

Maks λ

Kendala

$20000x_{11cv1} + 20000x_{12cv1} + 4000x_{11v2v1} + 4000x_{12v2v1} + 8500x_{11v3v1} + 8500x_{12v3v1} + 25000x_{11cv2} + 25000x_{12cv2} + 4000x_{11v1v2} + 4000x_{12v1v2} + 4500x_{11v3v2} + 4500x_{12v3v2} + 28500x_{11cv3} + +28500x_{12cv3} + 8500x_{11v1v3} + 8500x_{12v1v3} + 4500x_{11v2v3} + 4500x_{12v2v3} + 20000x_{11v1c} + 20000x_{11v1c} + 25000x_{11v2c} + 25000x_{12v2c} + 28500x_{11v3c} + 28500x_{12v3c} + 26000x_{11cv4} + 26000x_{12cv4} + 1000x_{11v5v4} + 1000x_{12v5v4} + 30000x_{11cv5} + 30000x_{12cv5} + 1000x_{11v4v5} + 1000x_{12v4v5} + 26000x_{11v4c} + 26000x_{12v4c} + 30000x_{11v5c} + 30000x_{12v5c} + 36000\lambda \leq 150000$

$20x_{11cv1} + 20x_{12cv1} + 8x_{11v2v1} + 8x_{12v2v1} + 15x_{11v3v1} + 15x_{12v3v1} + 24x_{11cv2} + 24x_{12cv2} + 8x_{11v1v2} + 8x_{12v1v2} + 9x_{11v3v2} + 9x_{12v3v2} + 37x_{11cv3} + +37x_{12cv3} + 15x_{11v1v3} + 15x_{12v1v3} + 9x_{11v2v3} + 9x_{12v2v3} + 20x_{11v1c} + 20x_{11v1c} + 24x_{11v2c} + 24x_{12v2c} + 37x_{11v3c} + 37x_{12v3c} + 26x_{11cv4} + 26x_{12cv4} + 4x_{11v5v4} + 4x_{12v5v4} + 28x_{11cv5} + 28x_{12cv5} + 4x_{11v4v5} + 4x_{12v4v5} + 26x_{11v4c} + 26x_{12v4c} + 28x_{11v5c} + 28x_{12v5c} + 41\lambda \leq 167$
$x_{11cv1} + x_{12cv1} + x_{11v2v1} + x_{12v2v1} + x_{11v3v1} + x_{12v3v1} = 1$
$x_{11cv2} + x_{12cv2} + x_{11v1v2} + x_{12v1v2} + x_{11v3v2} + x_{12v3v2} = 1$ (5)
$x_{11cv3} + x_{12cv3} + x_{11v1v3} + x_{12v1v3} + x_{11v2v3} + x_{12v2v3} = 1$

$x_{11cv4} + x_{12cv4} + x_{11v5v4} + x_{12v5v4} = 1$
$x_{11cv5} + x_{12cv5} + x_{11v4v5} + x_{12v4v5} = 1$
$x_{11cv1} + x_{12cv1} + x_{11cv2} + x_{12cv2} + x_{11cv3} + x_{12cv3} = 1$
$x_{11v2v1} + x_{12v2v1} + x_{11v1v2} + x_{12v1v2} + x_{11v1v3} + x_{12v1v3} = 1$
$x_{11v3v1} + x_{12v3v1} + x_{11v3v2} + x_{12v3v2} + x_{11v2v3} + x_{12v2v3} = 1$
$x_{11cv4} + x_{12cv4} + x_{11cv5} + x_{12cv5} = 1$
$x_{11v5v4} + x_{12v5v4} + x_{11v4v5} + x_{12v4v5} = 1$
$x_{11cv1} \leq 1$; $x_{12cv1} \leq 1$; $x_{11cv2} \leq 1$; $x_{12cv2} \leq 1$; $x_{11cv3} \leq 1$; $x_{12cv3} \leq 1$
$x_{11cv4} \leq 1$; $x_{12cv4} \leq 1$; $x_{11cv5} \leq 1$; $x_{12cv5} \leq 1$
$x_{11v1c} \leq 1$; $x_{12v1c} \leq 1$; $x_{11v2c} \leq 1$; $x_{12v2c} \leq 1$; $x_{11v3c} \leq 1$; $x_{12v3c} \leq 1$
$x_{11v4c} \leq 1$; $x_{12v4c} \leq 1$; $x_{11v5c} \leq 1$; $x_{12v5c} \leq 1$
$x_{11cv1} + x_{12cv1} + x_{11v2v1} + x_{12v2v1} + x_{11v3v1} + x_{12v3v1} - x_{11v1c} - x_{12v1c} - x_{11v1v2} - x_{12v1v2} - x_{11v1v3} - x_{12v1v3} = 0$
$x_{11cv2} + x_{12cv2} + x_{11v1v2} + x_{12v1v2} + x_{11v3v2} + x_{12v3v2} - x_{11v2c} - x_{12v2c} - x_{11v2v1} - x_{12v2v1} - x_{11v2v3} - x_{12v2v3} = 0$
$x_{11cv3} + x_{12cv3} + x_{11v1v3} + x_{12v1v3} + x_{11v2v3} + x_{12v2v3} - x_{11v3c} - x_{12v3c} - x_{11v3v1} - x_{12v3v1} - x_{11v3v2} - x_{12v3v2} = 0$
$x_{11cv3} + x_{12cv3} + x_{11v1v3} + x_{12v1v3} + x_{11v2v3} + x_{12v2v3} - x_{11v3c} - x_{12v3c} - x_{11v3v1} - x_{12v3v1} - x_{11v3v2} - x_{12v3v2} = 0$
$x_{11cv4} + x_{12cv4} + x_{11v5v4} + x_{12v5v4} - x_{11v4c} - x_{12v4c} - x_{11v4v5} - x_{12v4v5} = 0$
$x_{11cv5} + x_{12cv5} + x_{11v4v5} + x_{12v4v5} - x_{11v5c} - x_{12v5c} - x_{11v5v4} - x_{12v5v4} = 0$
$z_{1cv1} + z_{1v2v1} + z_{1v3v1} - z_{1v1c} - z_{1v1v2} - z_{1v1v3} = (60)^2(x_{11cv1} + x_{12cv1} + x_{11v2v1} + x_{12v2v1} + x_{11v3v1} + x_{12v3v1})$
$z_{1cv2} + z_{1v1v2} + z_{1v3v2} - z_{1v2c} - z_{1v2v1} - z_{1v2v3} = (60)^2(x_{11cv2} + x_{12cv2} + x_{11v1v2} + x_{12v1v2} + x_{11v3v2} + x_{12v3v2})$
$z_{1cv3} + z_{1v1v3} + z_{1v2v3} - z_{1v3c} - z_{1v3v1} - z_{1v3v2} = (45)^2(x_{11cv3} + x_{12cv3} + x_{11v1v3} + x_{12v1v3} + x_{11v2v3} + x_{12v2v3})$
$z_{1cv4} + z_{1v5v4} - z_{1v4c} - z_{1v4v5} = (90)^2(x_{11cv4} + x_{12cv4} + x_{11v5v4} + x_{12v5v4})$
$z_{1cv5} + z_{1v4v5} - z_{1v5c} - z_{1v5v4} = (45)^2(x_{11cv5} + x_{12cv5} + x_{11v4v5} + x_{12v4v5})$
$180 \leq ct_1 \leq 222$
$z_{1cv1} + z_{1cv2} + z_{1cv3} \leq 9225$
$z_{1cv4} + z_{1cv5} \leq 10125$
$x_{klij} \in \{0,1\}$; $z_{kij}, ct_1 \geq 0$; $\lambda \in [0,1]$

Penyelesaian model (5) menggunakan software Winqsb, diperoleh solusi sebagai berikut.

$\lambda = 1$; $x_{11cv1} = 1$; $x_{11v1v2}; x_{11v2v3} = 1$; $x_{11v3c} = 1$; $x_{11cv5} = 1$; $x_{11v5v4} = 1$; $x_{11v4c} = 1$; $z_{1cv1} = 3600$; $x_{11cv2} = 3600$; $x_{11cv3} = 2025$; $x_{11cv4} = 8100$; $x_{11cv5} = 2025$

Total biaya $Z_1 = 114000$; total waktu pengiriman Z_2 =132. Rute pengiriman optimal $c \to v_1 \to v_2 \to v_3 \to c$ dan $c \to v_5 \to v_4 \to c$. Total biaya untuk dua tour pengiriman sebesar Rp 114.000 dengan total waktu selama 132 menit.

Rute pengiriman untuk tour satu adalah distributor, pengecer 1, pengecer 2, pengecer 3 dan kembali lagi ke distributor. Rute pengiriman dua adalah distributor, pengecer 5, pengecer 4 dan kembali lagi ke distributor. Rute pengiriman yang dilakukan sebelumnya oleh distributor tidak tetap, total biaya rata-rata setiap tour sebesar Rp.59.000 atau sebesar Rp 118.000 untuk dua tour pengiriman. Menggunakan model FMOCIRP berdasarkan pembagian wilayah pada Gambar 1 diperoleh total biaya lebih minimum sebesar Rp 4.000 dibandingkan dengan tour yang dilakukan distributor sebelumnya.

4. KESIMPULAN

Berdasarkan hasil dan pembahasan dapat disimpulkan bahwa :

1. Pendekatan *fuzzy* dapat digunakan untuk penyelesaian model FMOCIRP pada permasalahan pendistribusian produk.
2. Pada permasalahan optimasi pendistribusian telur dengan 5 depot diperoleh optimal waktu pengiriman selama 132 menit dan total biaya sebesar Rp 114000 dengan rute optimal $c \to v_1 \to v_2 \to v_3 \to c$ dan $c \to v_5 \to v_4 \to c$.

5. SARAN

Pada penelitian ini hanya dipertimbangkan satu jenis kendaraan, untuk penelitian selanjutnya dapat dipertimbangkan beberapa jenis kendaraan dengan perbedaan kapasitas pengangkutan. Pada beberapa wilayah terdapat jalan dengan jalur satu arah, model yang dikembangkan pada penelitian ini mengasumsikan bahwa jalur yang dilalui adalah dua arah, untuk selanjutnya dapat ditambahkan beberapa kondisi pada model jika jalur yang dilalui adalah satu arah.

DAFTAR PUSTAKA

Azadeh, A., S. Elahi, M.Hosseinabadi Farahani, and B. Nasirian. 2017. "A Genetic Algorithm-Taguchi Based Approach for Inventory Routing Problem of a Single Perishable Product with Transshipment." *Computers & Industrial Engineering* 104:124–33. Retrieved (http://dx.doi.org/10.1016/j.cie.2016.12.019).

Azadi, Zahra, Sandra D. Eksioglu, Burak Eksioglu, and Gokce Palak. 2019. "Stochastic Optimization Models for Joint Pricing and Inventory Replenishment of Perishable Products." *Computers & Industrial Engineering* 127(January):625–42. Retrieved (https://doi.org/10.1016/j.cie.2018.11.004)

Bertazzi, Luca, Leandro C. Coelho, Annarita De Maio, and Demetrio Laganà. 2019. "A Matheuristic Algorithm for the Multi-Depot Inventory Routing Problem." *Transportation Research Part E* 122:524–44. Retrieved (https://doi.org/10.1016/j.tre.2019.01.005).

Chitsaz, Masoud, Ali Divsalar, and Pieter Vansteenwegen. 2016. "A Two Phase Algorithm For The Cyclic Inventory Routing Problem." *European Journal of Operational Research* 254(2, 16 October 2016):410–26. Retrieved (http://dx.doi.org/10.1016/j.ejor.2016.03.056).

Christos, Y. D. .., Maravelias Jose, and M. Pinto. 2017. "Solution Methods for Vehicle-Based Inventory Routing Problems." *Computers and Chemical Engineering* 101(9 June 2017):259–78. Retrieved (http://dx.doi.org/10.1016/j.compchemeng.2017.02.036).

Guimaraes, T. .., Leandro C. Coelho, M.Clede. Schenekemberg, and T. .. Scarpin. 2019. "The Two-Echelon Multi-Depot Inventory Routing Problem." *Computers and Operation Research* 101(January 2019):220–33.

Hu, Chaofang, Zelong Zhang, Na Yang, Hyo S. Shin, and Antonios Tsourdos. 2019. "Fuzzy Multiobjective Cooperative Surveillance of Multiple UAVs Based on Distributed Predictive Control for Unknown Ground Moving Target in Urban Environment Chaofang." *Aerospace Science and Technology* 84(January 2019):329–38. Retrieved (https://doi.org/10.1016/j.ast.2018.10.017).

Hu, Weihong, Alejandro Toriello, and Maged Dessouky. 2018. "Integrated Inventory Routing Problem and Freight Consolidation Perishable Goods." *European Journal of Operational Research* 271 (2, 1 December 2018): 548–60. Retrieved (https://doi.org/10.1016/j.ejor.2018.05.034)

Liang, Huijun, Yungang Liu, Fengzhong Li, and Yanjun Shen. 2018. "Electrical Power and Energy Systems A Multiobjective Hybrid Bat Algorithm for Combined Economic / Emission Dispatch." *Electrical Power and Energy Systems* 101(March):103–15. Retrieved (https://doi.org/10.1016/j.ijepes.2018.03.019).

Liu, Miao and Ze Dong. 2018. "Multiobjective Robust H 2 / H ∞ Fuzzy Tracking Control for Thermal System of Power Plant." *Journal of Process Control* 70:47–64. Retrieved (https://doi.org/10.1016/j.jprocont.2018.08.004).

Nikzad, Erfaneh, Mahdi Bashiri, and Fabricio Oliveira. 2018. "Two-Stage Stochastic Programming Approach For The Medical Drug Inventory Routing Problem Under Uncertainty." *Computers & Industrial Engineering* 128:358–70. Retrieved (https://doi.org/10.1016/j.cie.2018.12.055)

Palacios, Juan José, Inés González-rodríguez, Camino R. Vela, and Jorge Puente. 2017. "Robust Multiobjective Optimisation For Fuzzy Job Shop Problems." *Applied Soft Computing Journal* 56:604–16. Retrieved (http://dx.doi.org/10.1016/j.asoc.2016.07.004).

Rahimi, Mohammad, Armand Baboli, and Yacine Rekik. 2017. "Multi-Objective Inventory Routing Problem : A Stochastic Model to Consider Profit , Service Level and Green Criteria." *Transportation Research Part E* 101:59–83. Retrieved (http://dx.doi.org/10.1016/j.tre.2017.03.001).

Rau, Hsin, Syarif Daniel, and Gede Agus. 2018. "Optimization of the Multi-Objective Green Cyclical Inventory Routing Problem Using Discrete Multi-Swarm PSO Method." *Transportation Research Part E* 120(October):51–75. Retrieved (https://doi.org/10.1016/j.tre.2018.10.006).

Santos, Edcarllos, Luiz Satoru, and Luidi Simonetti. 2016. "A Hybrid Heuristic Based on Iterated Local Search for Multivehicle Inventory Routing Problem." *Electronic Notes in Discrete Mathematics* 52:197–204. Retrieved (http://dx.doi.org/10.1016/j.endm.2016.03.026).

Soysal, Mehmet. 2016. "Closed-Loop Inventory Routing Problem for Returnable Transport Items." *Transportation Research Part D* 48(December 2015):31–45. Retrieved (http://dx.doi.org/10.1016/j.trd.2016.07.001).

Susanti, Eka, Oki Dwipurwani, and Evi Yuliza. 2017. "Optimasi Kendaraan Pengangkut Sampah Menggunakan Model Fuzzy Goal Programming." *Jurnal Matematika* 7(2):119–23.